Collecting and Interpreting Quantitative Data

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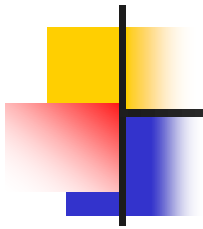
Overview



- Introduction
- Terms and fundamental concepts
- Tabular and graphical tools for describing data
- Numerical methods for describing and interpreting data
- MATLAB commands
- Summary

Introduction – Basic Questions

- How can we make assessment easier? – minimize the effort required to collect data
- How can we learn more from the assessment results we obtain? – use tools to interpret quantitative data

Making Assessment Easier

- Assess existing student work rather than creating or acquiring separate instruments.
- Measure only a sample of the population to be assessed.
- Depend on assessments at the College or University level.

Assessment Questions

After completing an assessment, one might ask:

- What do the assessment results mean?
- Was the sample used valid (representative and large enough)?
- Were the results obtained valid?
- Were the instrument and process utilized reliable?
- Is a difference between two results significant?

Fundamental Concepts

- Sampling a population
- Central tendency of data
- Frequency distribution of data
- Variance among data
- Correlation between data sets

Definition of Terms Related to Sampling

- **Data**: Observations (test scores, survey responses) that have been collected
- **Population**: Complete collection of all elements to be studied (e.g., all students in the program being assessed)
- **Sample**: Subset of elements selected from a population
- **Parameter**: A numerical measurement of a population
- **Statistic**: A numerical measurement describing some characteristic of a sample

Sampling Example

There are 1000 students in our program, and we want to study certain achievements of these students. A subset of 100 students is selected for measurements.

Population = 1000 students

Sample = 100 students

Data = 100 achievement measurements

Some Types of Sampling

- Random sample: Each member of a population has an equal chance of being selected
- Stratified sampling: The population is divided into sub-groups (e.g., male and female) and a sample from each sub-group is selected
- Convenience sampling: The results that are the easiest to get make up the sample

Problems with Sampling

- The sample may not be representative of the population.
- The sample may be too small to provide valid results.
- It may be difficult to obtain the desired sample.

Measure of Central Tendency: Mean

- n = number of observations in a sample
- $x_1, x_2, \ldots, x_n$ denotes these n observations
- $\bar{x}$, the sample **mean**, is the most common measure of center
- $\bar{x}$ (a statistic) is the arithmetic mean of the n observations:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

- μ represents the population mean, a parameter

Measure of Central Tendency: Median

- The **median** of a set of measurements is the middle value when the measurements are arranged in numerical order.
- If the number of measurements is even, the median is the mean of the two middle measurements.
  - Example: {1, 2, 3, 4, 5} Median = 3
  - Example: {1, 2, 3, 4, 100} Median = 3
  - Example: {1, 2, 3, 4, 5, 6} Median = (3 + 4)/2 = 3.5

Comparison of Mean and Median

- A survey of computer scientists yielded the following seven annual salaries:
  - $31.3K, $41K, $45.1K, $46.3K, $47.5K, $51.6K, $61.3K

median and mean salary

- If we add Bill Gates to the sample for this survey, the new sample (8 values) is:
  - $31.3K, $41K, $45.1K, $46.3K, $47.5K, $51.6K, $61.3K, $966.7K

median = $46.9K (slight increase)

mean = $161.35K (large increase)

- Outliers have a large effect on the mean, but not the median

Frequency Distribution of Data

The tabulation of raw data obtained by dividing the data into groups of some size and computing the number of data elements falling within each pair of group boundaries

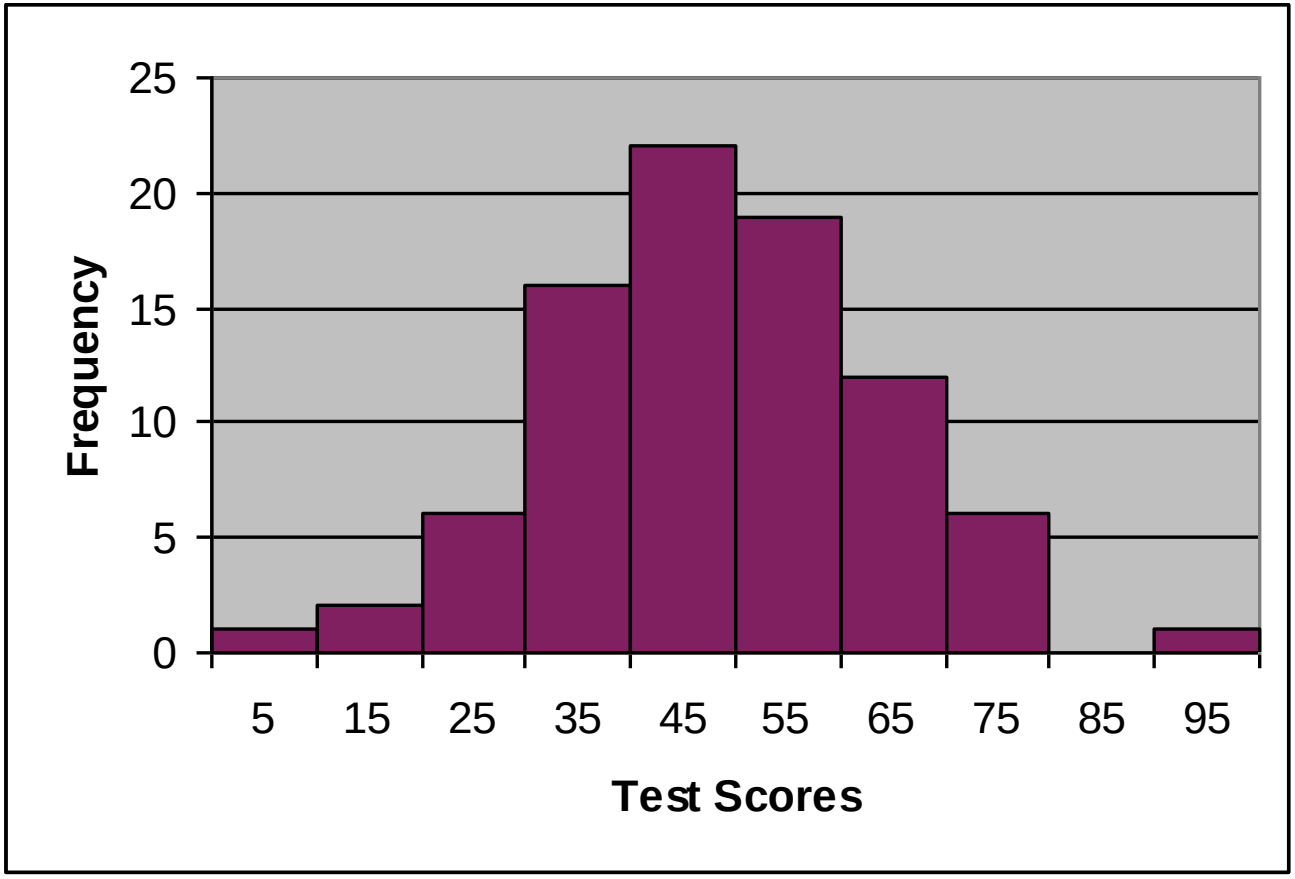
Frequency Distribution – Tabular Form

Group Interval	Frequency	Relative Frequency
0.00-9.99	1	1.18%
10.00-19.99	2	2.35%
20.00-29.99	6	7.06%
30.00-39.99	16	18.82%
40.00-49.99	22	25.88%
50.00-59.99	19	22.35%
60.00-69.99	12	14.12%
70.00-79.99	6	7.06%
80.00-89.99	0	0.00%
90.00-100.00	1	1.18%

Histogram

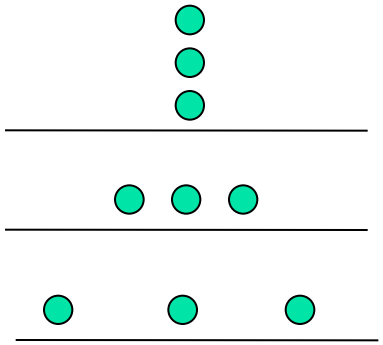
- A histogram is a graphical display of statistical information that uses rectangles to show the frequency of data items in successive numerical intervals of equal size. In the most common form of histogram, the independent variable is plotted along the horizontal axis and the dependent variable is plotted along the vertical axis.

Frequency Distribution -- Histogram



Variation among Data

- **The following three sets of data have a mean of 10:**
  - {10, 10, 10}
  - {5, 10, 15}
  - {0, 10, 20}



- **A numerical measure of their variation is needed to describe the data.**
- **The most commonly used measures of data variation are:**
  - Range
  - Variance
  - Standard Deviation

Measures of Variation: Variance

- **Sample of size n: x_1, x_2, …, x_n**
- **One measure of positive variation is** $(x_i - \bar{x})^2$
- **Definition of sample variance**

  (sample size = n): $$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$
- **Definition of population variance**

  (population size = N): $$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

Measures of Variation: Standard Deviation

- **Sample Standard Deviation:** $$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

- **Population Standard Deviation:** $$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$$

- The units of standard deviation are the same as the units of the observations

Measures of Variation: Variance and Standard Deviation

The following data sets each have a mean of 10.

Data Set	Variance	Standard Deviation
10, 10, 10	(0+0+0)/2 = 0	0
5, 10, 15	(25 + 0 + 25)/2 = 25	5
0, 10, 20	(100 + 0 + 100)/2 = 100	10

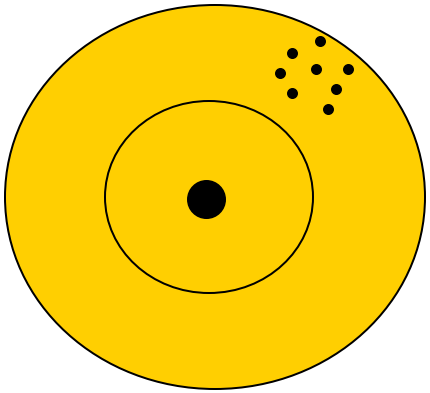
↑

Good measure of variation

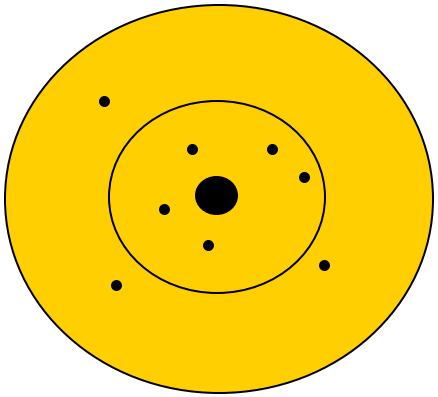
Reliability and Validity

- *Reliability* refers to the consistency of a number of measurements taken using the same measurement method on the same subject (i.e., how good are the operational metrics and the measurement data).
- *Validity* refers to whether the measurement really measures what it was intended to measure (i.e., the extent to which an empirical measure reflects the real meaning of the concept under consideration).

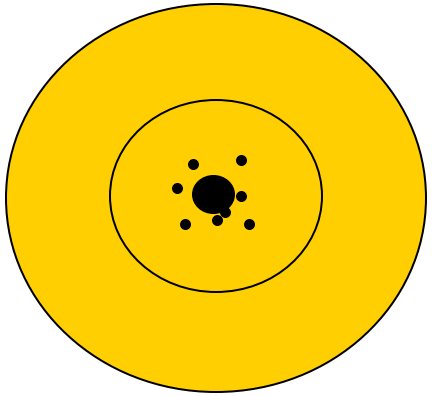
Reliability and Validity



Reliable but
not valid



Valid but
not reliable

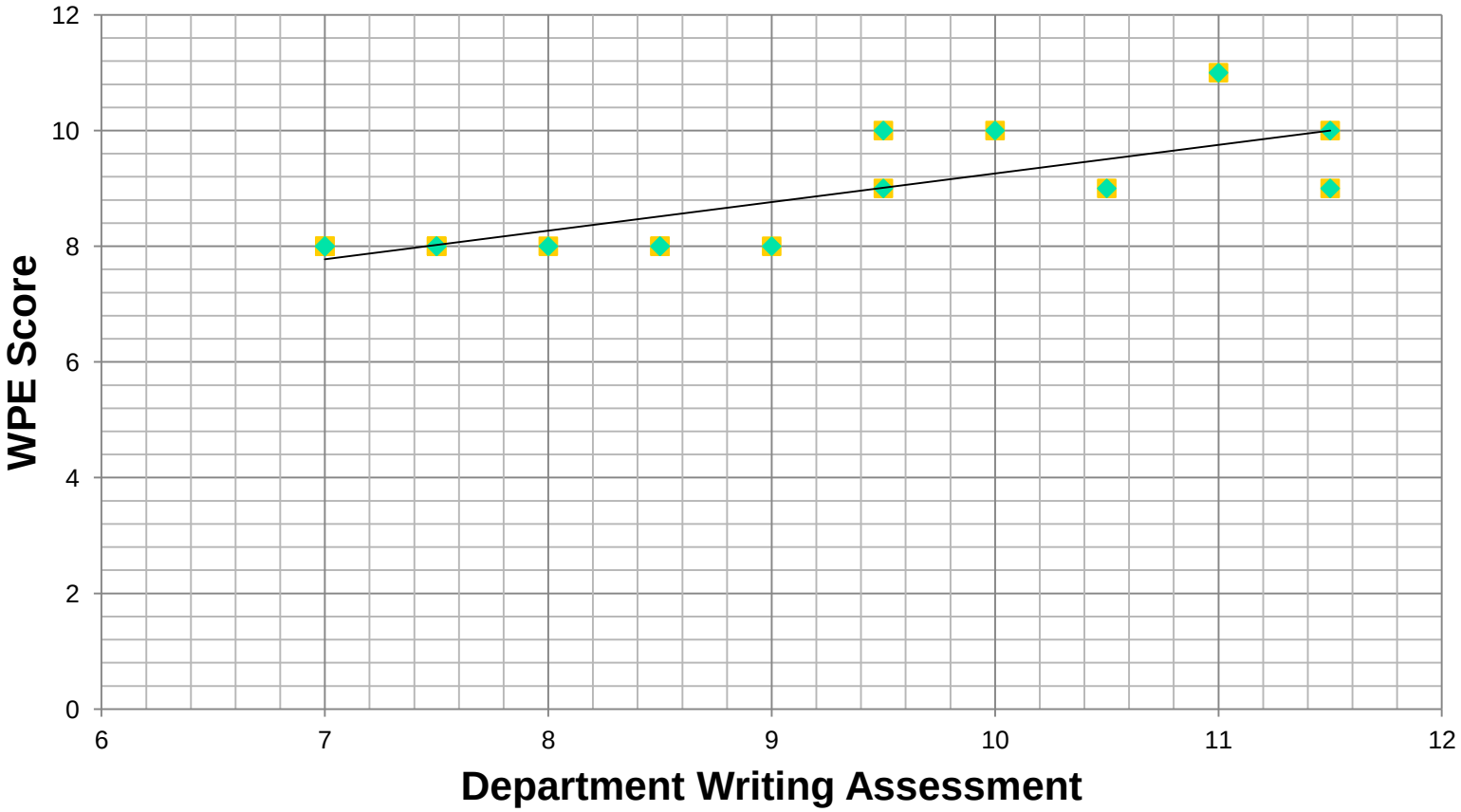


Reliable
& valid

Correlation

- Correlation is probably the most widely used statistical method to assess relationships among observational data.
- Correlation can show whether and how strongly two sets of observational data are related.
- This is one way to show validity by attempting to correlate the results from different approaches to assess the same outcome.

Example Correlation



Group Problem

- Assume your goal is to assess the written communication skills of students in your program. (Assume the number of students in the program is large and that you already have a rubric to use in assessing student writing.)
- Working with your group devise an approach to accomplish this task.

Group Problem (Cont'd)

- Specifically, who would you assess and what student produced work items would you evaluate, i.e., how would you construct an appropriate sample of students (or student work) to assess?
- Identify any concerns or potential difficulties with your plan, including issues of reliability or validity.
- What questions do you have regarding the interpretation of results once the assessment is completed?