HW 6: 6.2, 6.7, 6.11, 6.13, 6.16, 6.23

6.2 Lowest cost for insulation

In this case, we are looking for the thickness t that will give the same thermal resistance (or thermal insulation) for a given area.

The resistance is $$R = \frac{t}{k\,A}$$

The resistance ratio of a material to the best insulator is $$\frac{R}{R_r} = \frac{t}{t_r}\frac{k_r}{k}$$

If the resistances are to be equal, then the resistance ratio = 1 and $$\frac{t}{t_r} = \frac{k}{k_r}$$

The weight per unit area is $\frac{W}{A} = t\,\rho$ thus the weight ratio is $$\frac{W}{W_r} = \frac{t}{t_r}\frac{\rho}{\rho_r}$$

The cost per unit weight is $C = C_w\,W$ thus the cost ratio is $$\frac{C}{C_r} = \frac{C_w}{C_{wr}}\frac{W}{W_r}$$

The lowest thermal conductivity is glass fibers. If we use glass fibers as the reference material, we can see that other materials require greater thickness to provide the same insulation. The lowest cost for a given thermal insulation is the glass fibers. The lightest material to provide the same insulation is cork board (with wool or glass fibers very close). The thinnest material is the glass fibers.

Material	conductivity $W/(m\,°K)$	density kg/m^3	cost/weight \$/kg	t/t_r	W/W_r	C/C_r
glass fibers	0.035	220	0.75	1.00	1.00	1.00
plywood	0.109	590	0.20	3.11	8.35	2.23
wool	0.038	200	1.00	1.09	0.99	1.32
cork board	0.042	150	6.84	1.20	0.82	7.46

6.7 Plexiglas window

conductivity resistance: $R = \frac{L}{k\,A}$

resistance ratio: $\frac{R_{plex}}{R_{glass}} = \frac{L_{plex}}{L_{glass}}\frac{k_{glass}}{k_{plex}} = 1.0$

$$L_{plex} = \frac{k_{glass}}{k_{plex}} L_{glass} = \frac{0.810}{0.195} 0.125\,\text{in} = 0.519\,\text{in}$$

6.11 Cooling of steel plate

The convection coefficients for the various situations are:

$$h_{free} = 20 \frac{\text{watt}}{\text{m}^2\,{}^\circ\text{K}} = 1\text{x}$$

$$h_{forced} = 200 \frac{\text{watt}}{\text{m}^2\,{}^\circ\text{K}} = 10\text{x}$$

$$h_{mist} = 10{,}000 \frac{\text{watt}}{\text{m}^2\,{}^\circ\text{K}} = 500\text{x}$$

6.13 Heat transfer from hot ball

The radiation heat transfer can be stated as follows (assuming a 100% view factor and perfect emissivity).

$$Q_{rad} = \sigma\, A_{surface} \left(T_H^4 - T_L^4\right)$$

The surface area of a sphere is $A = 4\pi r^2 = \pi d^2$

Using numbers

$$Q_{rad} = 1.714 \times 10^{-9} \frac{\text{Btu/hr}}{\text{ft}^2\,{}^\circ\text{R}^4} \frac{\pi 1^2\,\text{in}^2}{144\,\text{in}^2/\text{ft}^2} \left(960^4 - 535^4\right) {}^\circ R^4$$

$$Q_{rad} = 28.7 \frac{\text{Btu}}{\text{hr}}$$

For convection, using an upper end of free convection of $h = 5 \frac{\text{Btu/hr}}{\text{ft}^2\,{}^\circ\text{F}}$

$$Q_{conv} = h\,A\,\Delta T = 5 \frac{\text{Btu/hr}}{\text{ft}^2\,{}^\circ\text{F}} \frac{\pi 1^2\,\text{in}^2}{144\,\text{in}^2/\text{ft}^2} (500 - 75)\,{}^\circ\text{F} = 46.4 \frac{\text{Btu}}{\text{hr}}$$

From Figure 6.11, using a high temperature of 500 °F, and a temperature differential of 425 °F, we find an equivalent convection coefficient of $3.4 \dfrac{\text{Btu/hr}}{\text{ft}^2\ °\text{F}}$.

These two convection coefficients are almost the same, and the heat transfer from radiation and convection are almost the same.

6.16 Suspended mass at end of a rod

The equation for the heat flow and storage in the mass is $Q_k - Q_h = m\,C_p\,\dot{T}_s$

The conduction through the rod is $Q_k = \dfrac{k\,A_R}{\ell}\left(T_0 - T_s\right)$

The convection from the mass is $Q_h = h\,A_s\left(T_s - T_\infty\right)$

Combining,

$$\frac{k\,A_R}{\ell}\left(T_0 - T_s\right) - h\,A_s\left(T_s - T_\infty\right) = m\,C_p\,\dot{T}_s$$

or

$$\left[m\,C_p\,D + \frac{k\,A_R}{\ell} + h\,A_s\right]T_s = \frac{k\,A_R}{\ell}\,T_0 + h\,A_s\,T_\infty$$

Therefore, the transfer function is

$$T_s = \frac{\dfrac{k\,A_R}{\ell}\,T_0 + h\,A_s\,T_\infty}{\left[m\,C_p\,D + \dfrac{k\,A_R}{\ell} + h\,A_s\right]}$$

Normalizing

$$T_s = \frac{\dfrac{1}{1+\dfrac{h\,A_s\,\ell}{k\,A_R}}\,T_0 + \dfrac{1}{1+\dfrac{k\,A_R}{h\,A_s\,\ell}}\,T_\infty}{\left[\dfrac{m\,C_p}{\dfrac{k\,A_R}{\ell} + h\,A_s}\,D + 1\right]}$$

The time constant is

$$\tau = \frac{m\,C_p}{\dfrac{k\,A_R}{\ell} + h\,A_s}$$

The steady-state temperature is

$$T_s(\infty) = \frac{1}{1+\dfrac{h\,A_s\,\ell}{k\,A_R}}\,T_0 + \frac{1}{1+\dfrac{k\,A_R}{h\,A_s\,\ell}}\,T_\infty$$

6.23 Transient response of window glass

The Biot number for the Plexiglas considering the outside convection is

$$N_b = \frac{h\,t}{k} = \frac{30\,\dfrac{\text{watt}}{\text{m}^2\,°\text{C}}\,0.0075\ \text{m}}{0.75\,\dfrac{\text{watt}}{\text{m}\,°\text{C}}} = 0.30$$

Since this Biot number is three times greater than 0.1, three nodes should be used.

The resistance associated with each node is

$$R_n = \frac{t/n}{k\,A} = \frac{0.0075\ \text{m}/3}{0.75\,\dfrac{\text{watt}}{\text{m}\,°\text{C}}\,0.88\ \text{m}^2} = 0.00379\,\frac{°\text{C}}{\text{watt}}$$

The convection resistance is

$$R_h = \frac{1}{h\,A} = \frac{1}{30\,\dfrac{\text{watt}}{\text{m}^2\,°\text{C}}\,0.88\ \text{m}^2} = 0.0379\,\frac{°\text{C}}{\text{watt}}$$

Since the convection resistance is 10 times the conduction resistance per node, then we should consider the convection resistance.

From Section 6.4.2, we can use the following differential equations.

$$\dot{T}_i = \frac{2\,n}{C_p\,M\,R_n}\left[-\left(1+\frac{R_n}{R_{hi}}\right)T_i + \frac{R_n}{R_{hi}}T_{i\infty} + T_a\right]$$

$$\dot{T}_a = \frac{n}{C_p\,M\,R_n}\left[-2\,T_a + T_i + T_b\right]$$

$$\dot{T}_b = \frac{n}{C_p\ M\ R_n}\left[-2\ T_b + T_a + T_o\right]$$

$$\dot{T}_o = \frac{2\ n}{C_p\ M\ R_n}\left[-\left(1+\frac{R_n}{R_{ho}}\right)T_o + \frac{R_n}{R_{ho}}T_{o\infty} + T_b\right]$$

For window glass, the density is 2800 kg/m³ , and the specific heat is 800 J/(kg °K).

$$M = \rho\, A\, t = 18.48\ \text{kg} \qquad \text{and} \qquad C_p\ M = 14{,}784\ \frac{\text{watt s}}{{}^\circ\text{K}}$$

$$\frac{n}{C_p\ M\ R_n} = 0.0535\ \frac{1}{\text{s}} \qquad \text{and} \qquad \frac{R_n}{R_h} = 0.10$$

Using these numbers, the differential equations can be stated as follows.

$$\dot{T}_i = 0.1071\frac{1}{\text{s}}\left[-(1.1)\ T_i + 0.1\ T_{i\infty} + T_a\right]$$

$$\dot{T}_a = 0.0535\frac{1}{\text{s}}\left[-2\ T_a + T_i + T_b\right]$$

$$\dot{T}_b = 0.0535\frac{1}{\text{s}}\left[-2\ T_b + T_a + T_o\right]$$

$$\dot{T}_o = 0.1071\frac{1}{\text{s}}\left[-(1.1)\ T_o + 0.1\ T_{o\infty} + T_b\right]$$

In order to solve this system, we must first determine the initial conditions from the steady-state conditions. You can either use digital simulation to determine the steady-state temperatures, or you can solve the 4 algebraic equations resulting from setting the derivatives equal to zero from the differential equations. The algebraic equations from the steady-state differential equations are given below along with the solution.

$$\begin{aligned} 0 &= -(1.1)\ T_i + 0.1(25) + T_a \\ 0 &= -2\ T_a + T_i + T_b \\ 0 &= -2\ T_b + T_a + T_o \\ 0 &= -(1.1)\ T_o + 0.1(20) + T_b \end{aligned} \qquad \begin{aligned} T_{i\infty} &= 25.000\,{}^\circ C \\ T_i &= 22.826\,{}^\circ C \\ T_a &= 22.609\,{}^\circ C \\ T_b &= 22.391\,{}^\circ C \\ T_o &= 22.174\,{}^\circ C \\ T_{o\infty} &= 20.000\,{}^\circ C \end{aligned}$$

Using these initial conditions and subjecting the outside temperature to a sudden change from 20 degrees to 15 degrees, we can get the following solutions for the temperatures of this system.