Determine a state variable representation for the following transfer functions using the ss function;

$$(a) G(s) = \frac{1}{s+25}$$

(b) 
$$G(s) = \frac{3s^2 + 10s + 3}{s^2 + 8s + 5}$$

(c) 
$$G(s) = \frac{s+10}{s^3+3s^2+3s+1}$$

Determine a transfer function representation for the following state variable models using the tf function:

(a) 
$$A = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$$
;  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ;  $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ 

(b) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 0 & 4 \\ 6 & 2 & 10 \end{bmatrix}; B = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

(c) 
$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$
;  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ;  $C = \begin{bmatrix} -2 & 1 \end{bmatrix}$ 

For the simple pendulum shown in Figure CP2.7, the nonlinear equation of motion is given by

$$\ddot{\theta}(t) + \frac{g}{L}\sin\theta = 0,$$

where L = 0.5 m, m = 1 kg, and g = 9.8 m/s<sup>2</sup>. When the nonlinear equation is linearized about the equilibrium point  $\theta = 0$ , we obtain the linear time-invariant model,

$$\ddot{\theta} + \frac{g}{L}\theta = 0.$$

Create an m-file to plot both the nonlinear and the linear response of the simple pendulum when the initial angle of the pendulum is  $\theta(0) = 30^{\circ}$  and explain any differences.

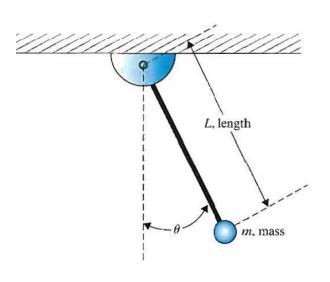


FIGURE CP2.7 Simple pendulum.

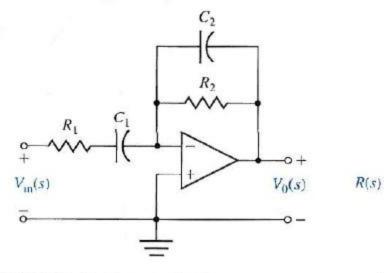
A system has a transfer function

$$\frac{X(s)}{R(s)} = \frac{(15/z)(s+z)}{s^2+3s+15}.$$

Plot the response of the system when R(s) is a unit step for the parameter z = 3, 6, and 12.

Consider the circuit shown in Figure CP3.3. Determine the transfer function  $V_0(s)/V_{in}(s)$ . Assume an ideal op-amp.

- (a) Determine the state variable representation when  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $C_1 = 0.5 \text{ mF}$ , and  $C_2 = 0.1 \text{ mF}$ .
- (b) Using the state variable representation from part (a), plot the unit step response with the step function.



Consider the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}.$$

- (a) Using the function, determine the transfer function Y(s)/U(s).
- (b) Plot the response of the system to the initial condition  $\mathbf{x}(0) = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}^T$  for  $0 \le t \le 10$ .

Using MATLAB function ode23 obtain the numerical solution for the differential equation given by

$$\frac{d^2\theta}{dt^2} + \frac{B}{m}\frac{d\theta}{dt} + \frac{g}{l}\sin\theta = 0$$

Where  $m = 0.5 \,\text{Kg}$ ,  $l = 0.613 \,\text{m}$ ,  $B = 0.05 \,\text{Kg-s/m}$ , and  $g = 9.81 \,\text{m/s}^2$ .

The initial angle at time t = 0 is  $\theta(0) = 0.5$  and  $\theta(0) = 0$ .

2.27 Magnetic levitation trains provide a high-speed, very low friction alternative to steel wheels on steel rails. The train floats on an air gap as shown in Figure P2.27 [27]. The levitation force F<sub>L</sub> is controlled by the coil current i in the levitation coils and may be approximated by

$$F_L=k\frac{i^2}{z^2},$$

where z is the air gap. This force is opposed by the downward force F = mg. Determine the linearized relationship between the air gap z and the controlling current near the equilibrium condition.

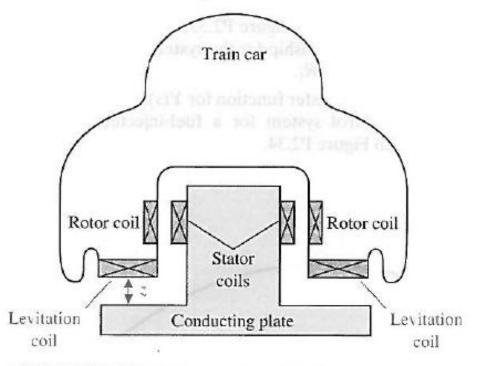


FIGURE P2.27 Cutaway view of train.