HW: Problem 1

Determine a state variable representation for the following transfer functions using the ss function;

$$(a)\ G(s) = \frac{1}{s+25}$$

$$(b)\ G(s) = \frac{3s^2+10s+3}{s^2+8s+5}$$

$$(c)\ G(s) = \frac{s+10}{s^3+3s^2+3s+1}$$

HW: Problem 1 Solution

The m-file script to compute the state-space models using the ss function is shown in Figure CP3.1.

```
% Part(a)
num = [1]; den = [1 25];
sys = tf(num,den);
sys_ss = ss(sys)
% Part(b)
num = [3 10 3]; den = [1 8 5];
sys = tf(num,den);
sys_ss = ss(sys)
% Part(c)
num = [1 10]; den = [1 3 3 1];
sys = tf(num,den);
sys_ss = ss(sys)
```

Part (a) output:

```
a =
     x1
 x1  -25

b =
     u1
 x1  1

c =
     x1
 y1  1

d =
     u1
 y1  0
```

Part (b) output:

```
a =
      x1    x2
 x1   -8  -2.5
 x2    2     0

b =
     u1
 x1  4
 x2  0

c =
      x1    x2
 y1 -3.5  -1.5

d =
     u1
 y1  3
```

Part (c) output:

```
a =
      x1    x2     x3
 x1   -3  -1.5  -0.25
 x2    2     0      0
 x3    0     2      0

b =
     u1
 x1  2
 x2  0
 x3  0

c =
      x1    x2    x3
 y1    0  0.25  1.25

d =
     u1
 y1  0
```

FIGURE CP3.1
Script to compute state-space models from transfer functions.

HW: Problem 1 Solution

For example, in part (c) the state-space model is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u \ ,$$

where $\mathbf{D} = [0]$ and

$$\mathbf{A} = \begin{bmatrix} -3 & -1.5 & -0.25 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0.25 & 1.25 \end{bmatrix}$$

HW: Problem 2

Determine a transfer function representation for the following state variable models using the tf function:

$$(a)\ A=\begin{bmatrix}0 & 1\\ 2 & 4\end{bmatrix};\ B=\begin{bmatrix}0\\ 1\end{bmatrix}; C=[1\ 0]$$

$$(b)\ A=\begin{bmatrix}1 & 1 & 0\\ -2 & 0 & 4\\ 6 & 2 & 10\end{bmatrix}; B=\begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}; C=[0\ 1\ 0]$$

$$(c)\ A=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix};\ B=\begin{bmatrix}0\\ 1\end{bmatrix}; C=[-2\ 1]$$

HW: Problem 2 Solution

The m-file script to compute the transfer function models using the **tf** function is shown in Figure CP3.2.

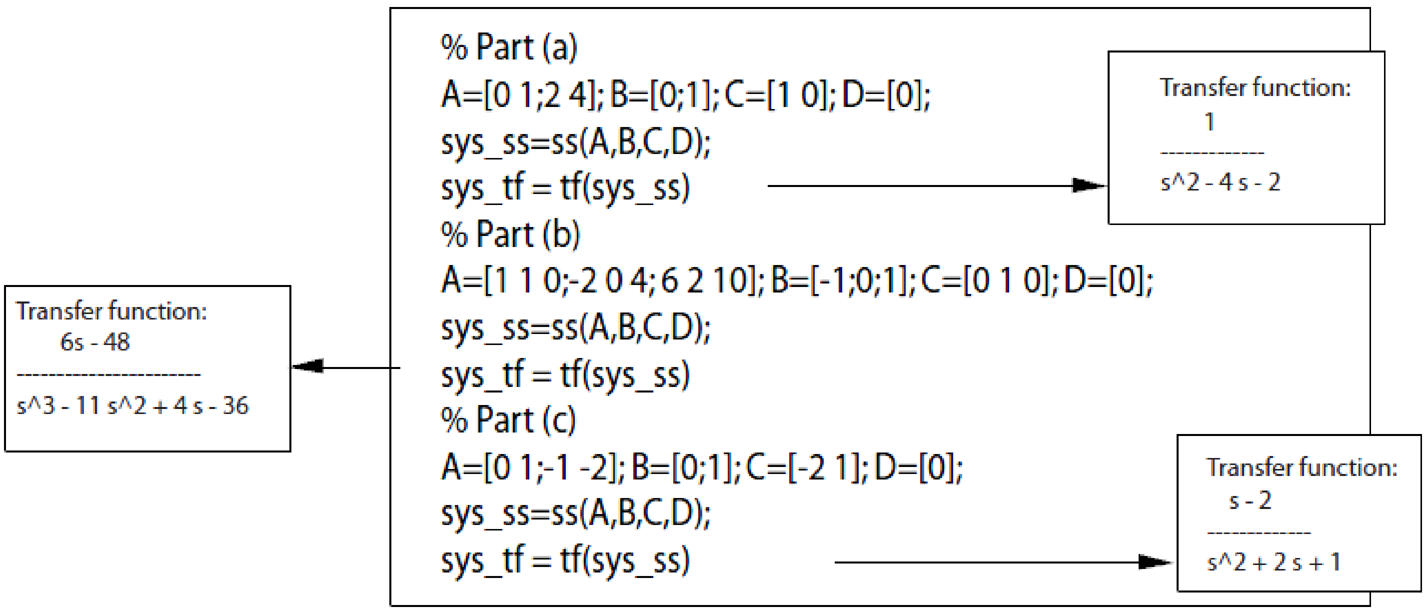


FIGURE CP3.2
Script to compute transfer function models from the state-space models.

HW: Problem 3

For the simple pendulum shown in Figure CP2.7, the nonlinear equation of motion is given by

$$\ddot{\theta}(t) + \frac{g}{L}\sin\theta = 0,$$

where $L = 0.5$ m, $m = 1$ kg, and $g = 9.8$ m/s^2. When the nonlinear equation is linearized about the equilibrium point $\theta = 0$, we obtain the linear time-invariant model,

$$\ddot{\theta} + \frac{g}{L}\theta = 0.$$

Create an m-file to plot both the nonlinear and the linear response of the simple pendulum when the initial angle of the pendulum is $\theta(0) = 30°$ and explain any differences.

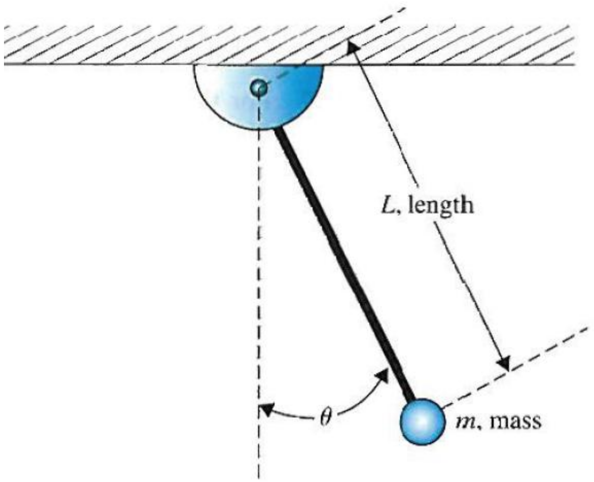


FIGURE CP2.7
Simple pendulum.

HW: Problem 3 Solution

The m-file script and plot of the pendulum angle is shown in Figure CP2.7. With the initial conditions, the Laplace transform of the linear system is

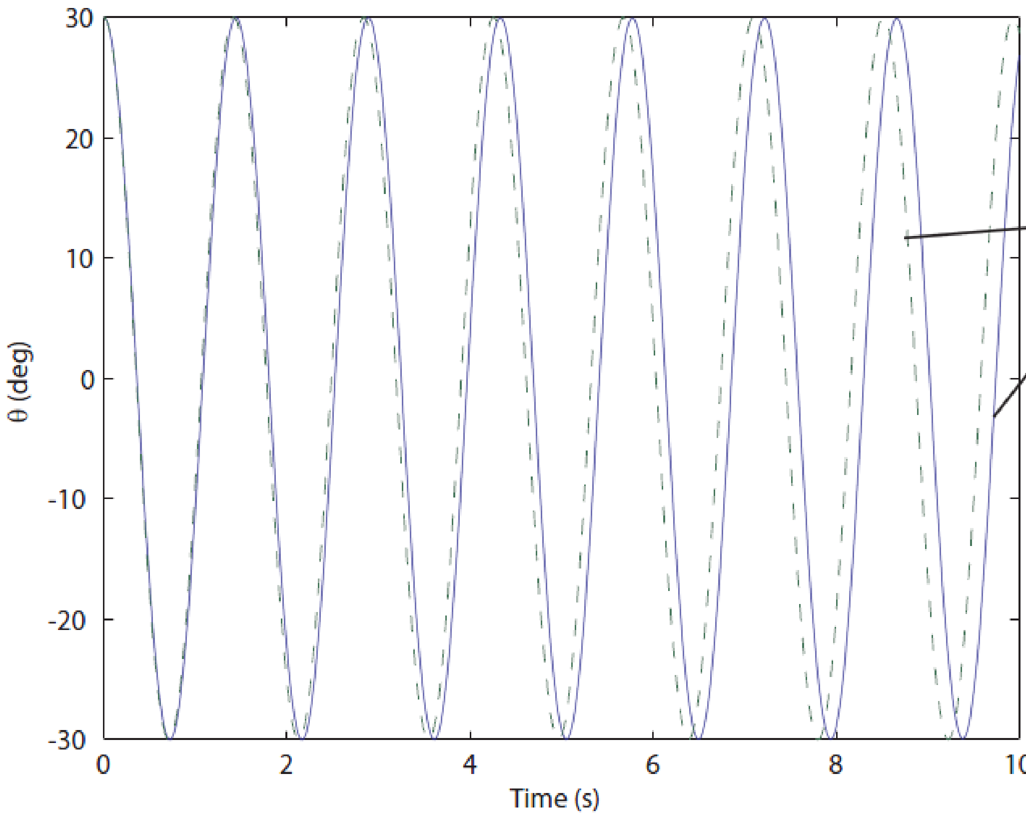
$$\theta(s) = \frac{\theta_0 s}{s^2 + g/L}.$$

To use the step function with the m-file, we can multiply the transfer function as follows:

$$\theta(s) = \frac{s^2}{s^2 + g/L}\frac{\theta_0}{s},$$

which is equivalent to the original transfer function except that we can use the step function input with magnitude θ_0. The nonlinear response is shown as the solid line and the linear response is shown as the dashed line. The difference between the two responses is not great since the initial condition of $\theta_0 = 30^\circ$ is not that large.

HW: Problem 3 Solution




```
L=0.5; m=1; g=9.8;
theta0=30;
% Linear simulation
sys=tf([1 0 0],[1 0 g/L]);
[y,t]=step(theta0*sys,[0:0.01:10]);
% Nonlinear simulation
[t,ynl]=ode45(@pend,t,[theta0*pi/180 0]);
plot(t,ynl(:,1)*180/pi,t,y,'--');
xlabel('Time (s)')
ylabel('\theta (deg)')
```

```
function [yd]=pend(t,y)
L=0.5; g=9.8;
yd(1)=y(2);
yd(2)=-(g/L)*sin(y(1));
yd=yd';
```

FIGURE CP2.7

Plot of θ versus xt when $\theta_0 = 30^\circ$.

HW: Problem 4

A system has a transfer function

$$\frac{X(s)}{R(s)} = \frac{(15/z)(s+z)}{s^2 + 3s + 15}.$$

Plot the response of the system when $R(s)$ is a unit step for the parameter $z = 3, 6,$ and 12.

HW: Problem 4 Solution

The system step responses for $z = 3, 6,$ and 12 are shown in Figure CP2.8.

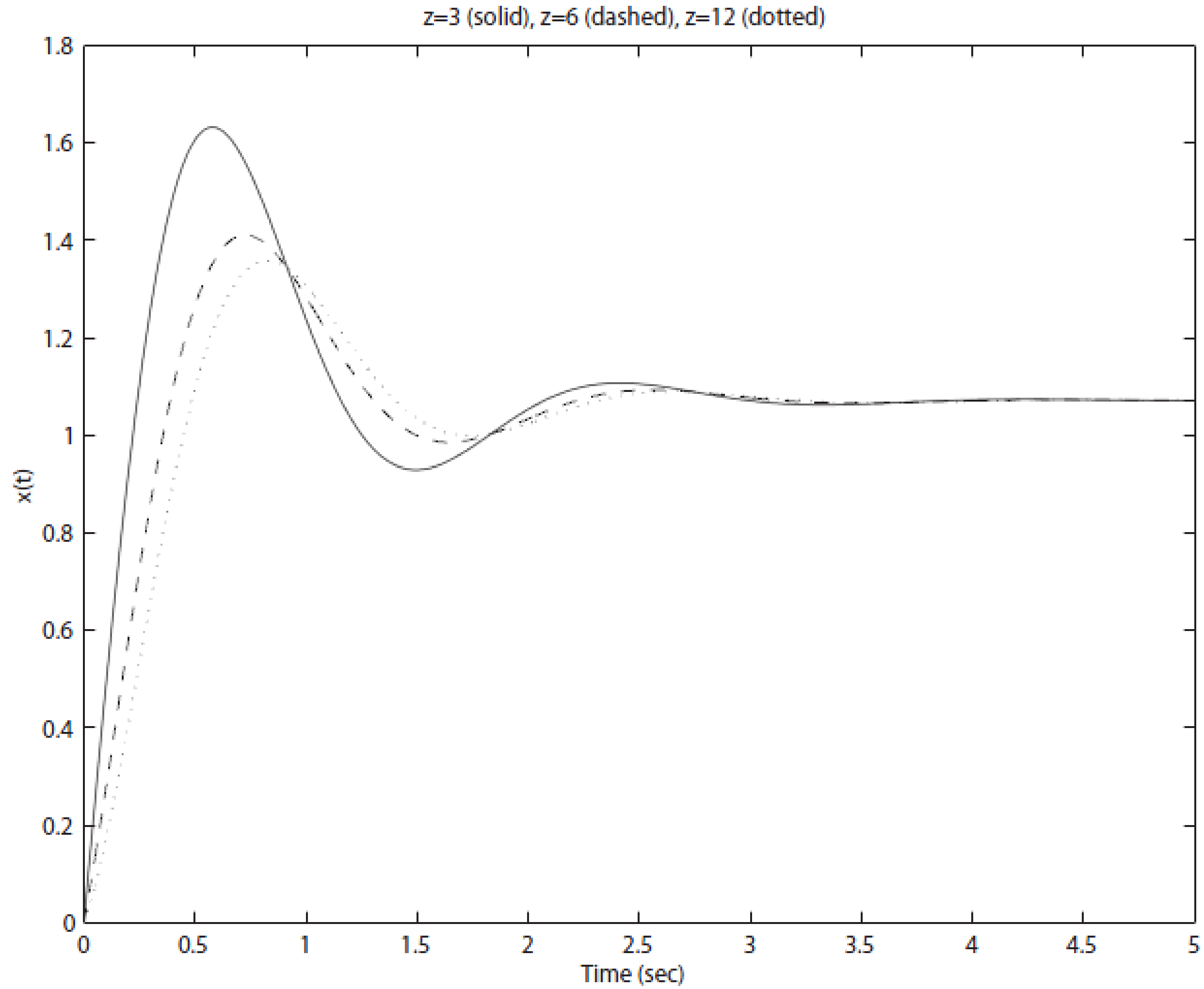


FIGURE CP2.8
The system response.

HW: Problem 5

Consider the circuit shown in Figure CP3.3. Determine the transfer function $V_0(s)/V_{in}(s)$. Assume an ideal op-amp.

(a) Determine the state variable representation when $R_1 = 1\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, $C_1 = 0.5$ mF, and $C_2 = 0.1$ mF.

(b) Using the state variable representation from part (a), plot the unit step response with the step function.

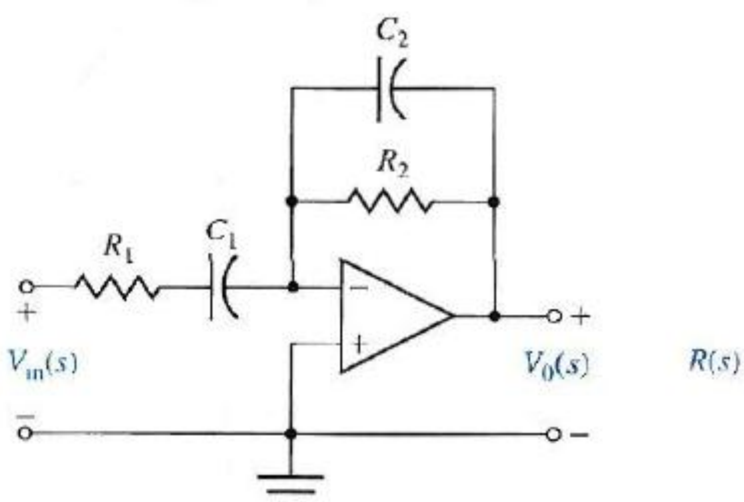


FIGURE CP3.3 An op-amp circuit.

HW: Problem 5 Solution

For an ideal op-amp, the voltage gain (as a function of frequency) is

$$V_o(s) = -\frac{Z_2(s)}{Z_1(s)} V_{in}(s),$$

where

$$Z_1 = R_1 + \frac{1}{C_1 s}$$

$$Z_2 = \frac{R_2}{1 + R_2 C_2 s}$$

are the respective circuit impedances. Therefore, we obtain

$$V_o(s) = -\left[\frac{R_2 C_1 s}{(1 + R_1 C_1 s)(1 + R_2 C_2 s)}\right] V_{in}(s).$$

The m-file script and step response is shown in Figure CP3.3.

HW: Problem 5 Solution

```
R1=1000; R2=10000; C1=0.0005; C2=0.0001;
numg=[R2*C1 0];
deng=conv([R1*C1 1],[R2*C2 1]);
 sys_tf=tf(numg,deng)
% Part (a)
%
sys_ss=ss(sys_tf)
% Part (b)
%
step(sys_ss)
```

```
a =
            x1          x2
      x1   -3.00000   -1.00000
      x2    2.00000          0
b =
            u1
      x1    4.00000
      x2          0
c =
            x1          x2
      y1    2.50000          0
d =
            u1
      y1          0

Continuous-time system.
```

HW: Problem 5 Solution

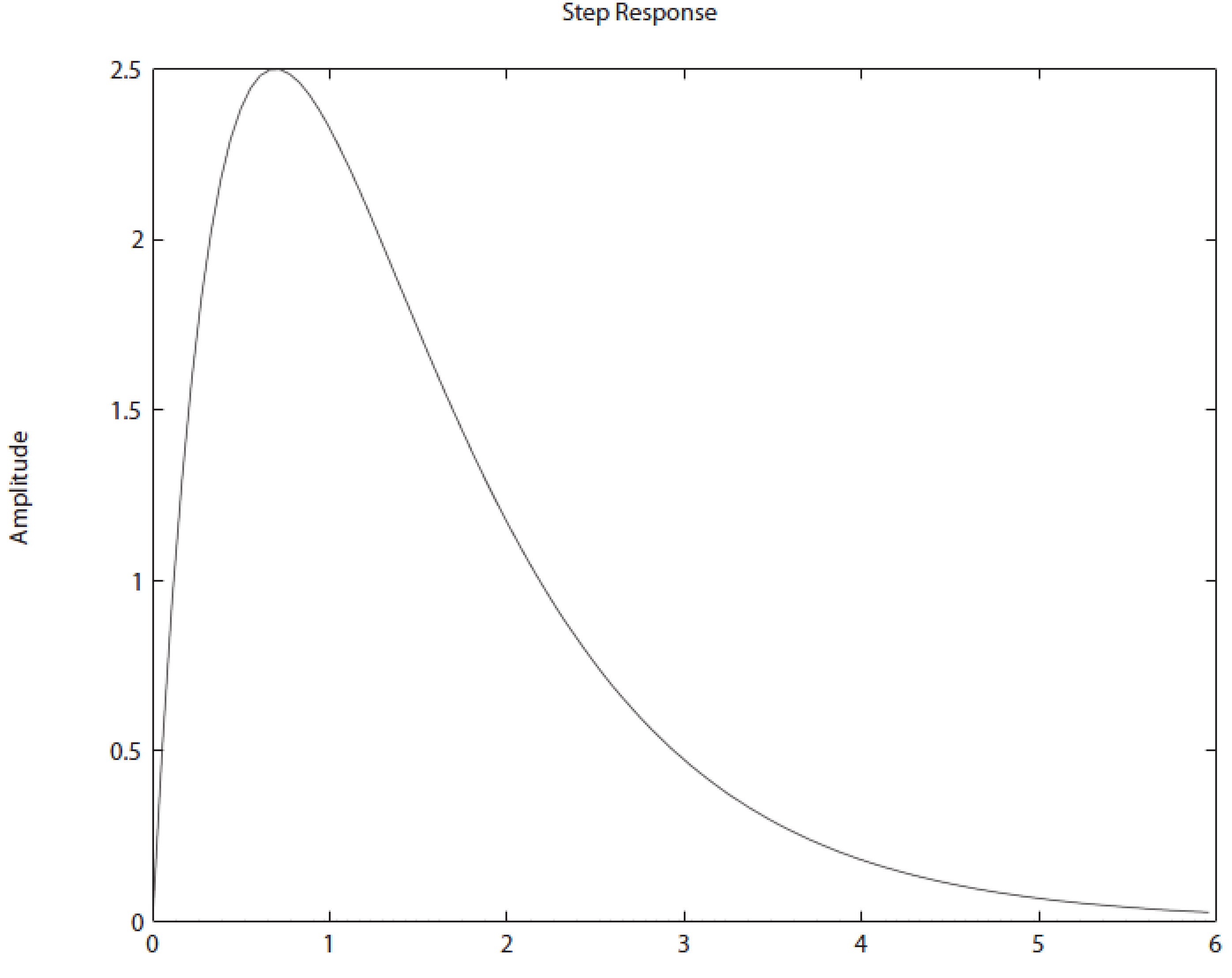


FIGURE CP3.3
The m-file script using the **step** function to determine the step response.

HW: Problem 6

Consider the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u,$$

$$y = [1 \quad 0 \quad 0]\mathbf{x}.$$

(a) Using the tf function, determine the transfer function $Y(s)/U(s)$.

(b) Plot the response of the system to the initial condition $\mathbf{x}(0) = [0 \quad -1 \quad 1]^T$ for $0 \leq t \leq 10$.

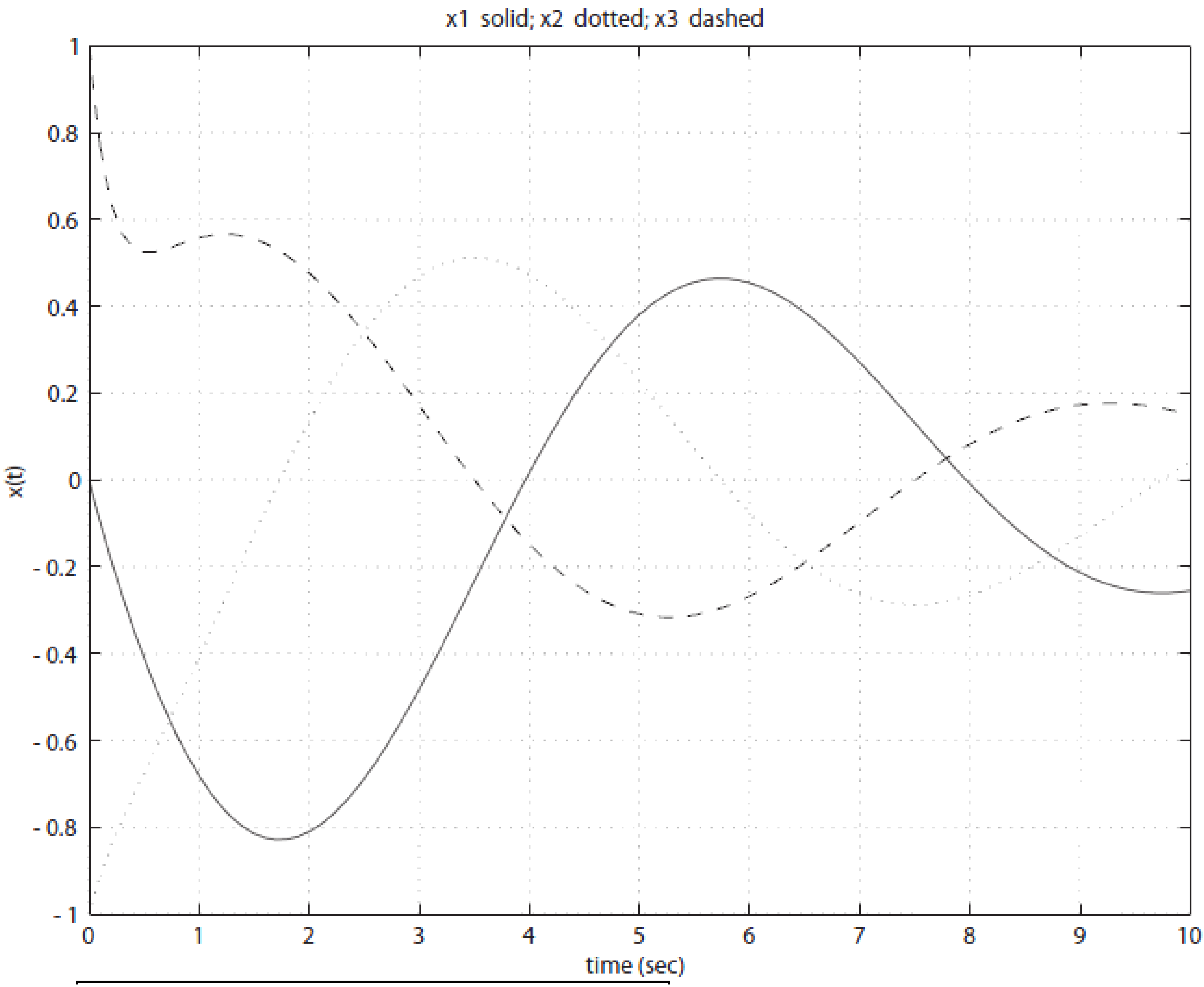
HW: Problem 6 Solution

The m-file script and state history is shown in Figure CP3.4. The transfer function equivalent is

$$G(s) = \frac{1}{s^3 + 5s^2 + 2s + 3} \ .$$

The computed state vector at $t = 10$ is the same using the simulation and the state transition matrix.

HW: Problem 6 Solution



HW: Problem 6 Solution

```
a=[0 1 0; 0 0 1; -3 -2 -5];
b=[0;0;1];
c=[1 0 0];
d=[0];
%
% Part (a)
%
sys_ss = ss(a,b,c,d)
sys_tf = tf(sys_ss)
%
% Part (b)
%
x0 = [0 -1 1];
t = [0:0.1:10];
u = 0*t;
[y,t,x] = lsim(sys_ss,u,t,x0);
plot(t,x(:,1),t,x(:,2),':',t,x(:,3),'--');
xlabel('time (sec)'), ylabel('x(t)'), grid
title('x1 - solid; x2 - dotted; x3 - dashed')
xf_sim = x(length(t),:)'
%
% Part (c)
%
dt = 10;
Phi = expm(a*dt);
xf_phi = Phi*x0'
```

```
Transfer function:
        1
---------------------
s^3 + 5 s^2 + 2 s + 3
```

```
xf_sim =

  -0.2545
   0.0418
   0.1500
```

```
xf_phi =

  -0.2545
   0.0418
   0.1500
```

FIGURE CP3.4
The m-file script using the **lsim** function to determine the step response.

HW: Problem 7

Using MATLAB function **ode23** obtain the numerical solution for the differential equation given by

$$\frac{d^2\theta}{dt^2} + \frac{B}{m}\frac{d\theta}{dt} + \frac{g}{l}\sin\theta = 0$$

Where $m = 0.5$ Kg, $l = 0.613$ m, $B = 0.05$ Kg-s/m, and $g = 9.81$ m/s^2.

The initial angle at time $t = 0$ is $\theta(0) = 0.5$ and $\dot{\theta}(0) = 0$.

HW: Problem 7 Solution

First we write the above equation in state variable form. Let $x_1 = \theta$, and $x_2 = \dot{\theta}$ (angular velocity), then

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{B}{m}x_2 - \frac{g}{l}\sin x_1$$

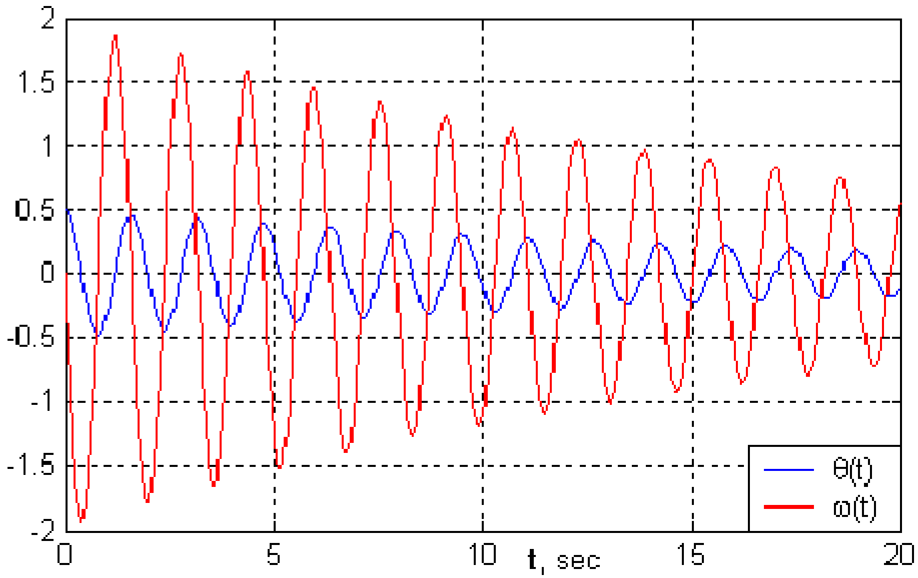
The above equations are defined in a function file named pendulumeq.m as follows

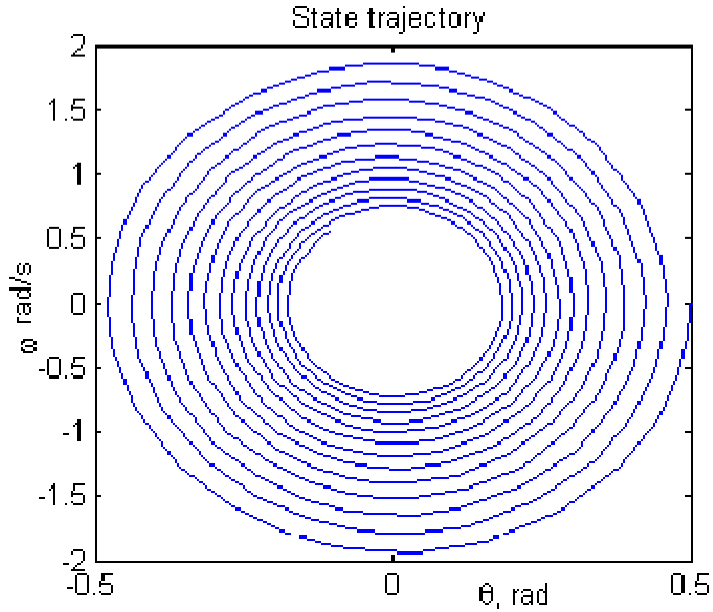
```
function xdot = pendulumeq(t, x);  % Returns the state derivative
m = 0.5; l = 0.613; B = 0.05; g = 9.81;
xdot = [x(2); -B/m*x(2)-g/l*sin(x(1))];
```

In a separate file named Lab3ExB4.m, the MATLAB function ode23 is used to obtain the solution of the given differential equations (defined in the file pendulumeq.m from 0 to 20 seconds.

HW: Problem 7 Solution


```
tspan = [0, 20];                          % time interval
x0 = [0.5; 0];                            % initial condition
[t, x] = ode23('pendulumeq', tspan, x0);
theta = x(:, 1);omega = x(:, 2);
figure(1), plot(t, theta, 'b', t, omega, 'r'), grid
xlabel('t, sec'), legend('\theta(t)', '\omega(t)')
figure(2), plot(theta, omega);
xlabel('\theta, rad'), ylabel('\omega rad/s')
title('State trajectory')
```





HW: Problem 8

P2.27 Magnetic levitation trains provide a high-speed, very low friction alternative to steel wheels on steel rails. The train floats on an air gap as shown in Figure P2.27 [27]. The levitation force F_L is controlled by the coil current i in the levitation coils and may be approximated by

$$F_L = k\frac{i^2}{z^2},$$

where z is the air gap. This force is opposed by the downward force $F = mg$. Determine the linearized relationship between the air gap z and the controlling current near the equilibrium condition.

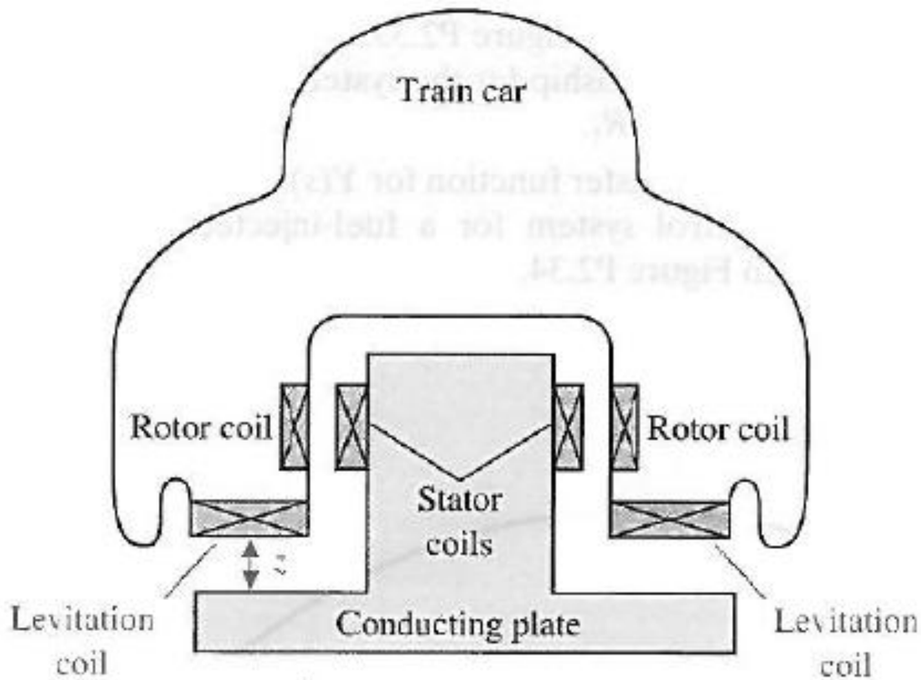


FIGURE P2.27 Cutaway view of train.

HW: Problem 9 Solution

The describing equation of motion is

$$m\ddot{z} = mg - k\frac{i^2}{z^2} \ .$$

Defining

$$f(z, i) = g - \frac{ki^2}{mz^2}$$

leads to

$$\ddot{z} = f(z, i) \ .$$

The equilibrium condition for i_o and z_o, found by solving the equation of motion when

$$\dot{z} = \ddot{z} = 0 \ ,$$

is

$$\frac{ki_o^2}{mg} = z_o^2 \ .$$

We linearize the equation of motion using a Taylor series approximation. With the definitions

HW: Problem 9 Solution

$$\Delta z = z - z_o \quad \text{and} \quad \Delta i = i - i_o \ ,$$

we have $\dot{\Delta} z = \dot{z}$ and $\ddot{\Delta} z = \ddot{z}$. Therefore,

$$\ddot{\Delta} z = f(z,i) = f(z_o,i_o) + \left.\frac{\partial f}{\partial z}\right|_{\substack{z=z_o \\ i=i_o}} \Delta z + \left.\frac{\partial f}{\partial i}\right|_{\substack{z=z_o \\ i=i_o}} \Delta i + \cdots$$

But $f(z_o,i_o) = 0$, and neglecting higher-order terms in the expansion yields

$$\ddot{\Delta} z = \frac{2ki_o^2}{mz_o^3}\Delta z - \frac{2ki_o}{mz_o^2}\Delta i \ .$$

Using the equilibrium condition which relates z_o to i_o, we determine that

$$\ddot{\Delta} z = \frac{2g}{z_o}\Delta z - \frac{g}{i_o}\Delta i \ .$$

Taking the Laplace transform yields the transfer function (valid around the equilibrium point)

$$\frac{\Delta Z(s)}{\Delta I(s)} = \frac{-g/i_o}{s^2 - 2g/z_o} \ .$$