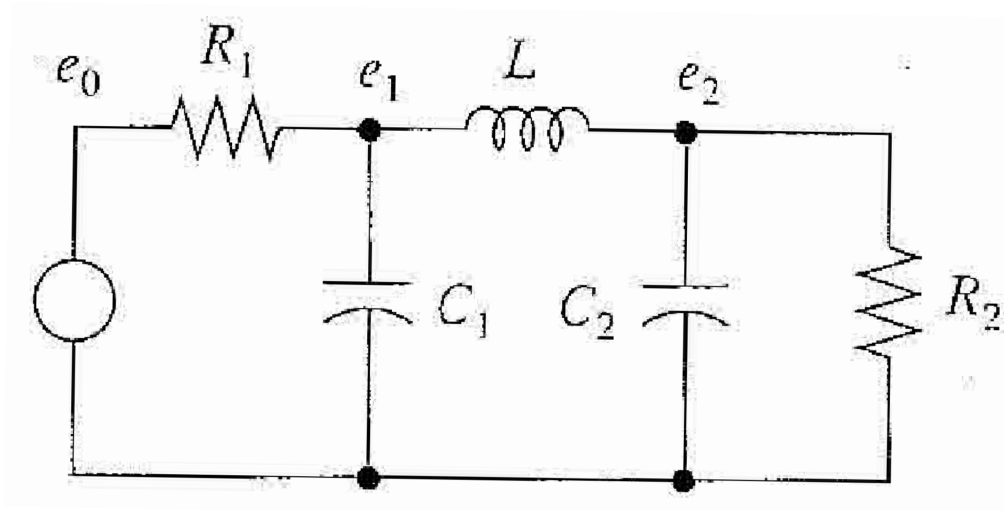
Techniques for Passive Circuit Analysis for State Space Differential Equations

1. Draw circuit schematic and label components (e.g., R_1, R_2, C_1, L_1…)
2. Assign voltage at each node (e.g., e_1, e_2)
3. Assign current in each component (e.g., i_1, i_2, ..) and show positive current direction with arrows
4. Write equation for current for each component (e.g., $i_{R1} = (e_1-e_2)/R_1$ or $i_{C1} = CDe_1$)
5. Write node equations for each significant node (not connected to voltage or current source)
6. Use capacitor voltages and inductor currents as state variables, rearrange component equations in first-order form. Use remaining component and node equations to reduce differential equations so that they contain only state variables and input voltage or current sources

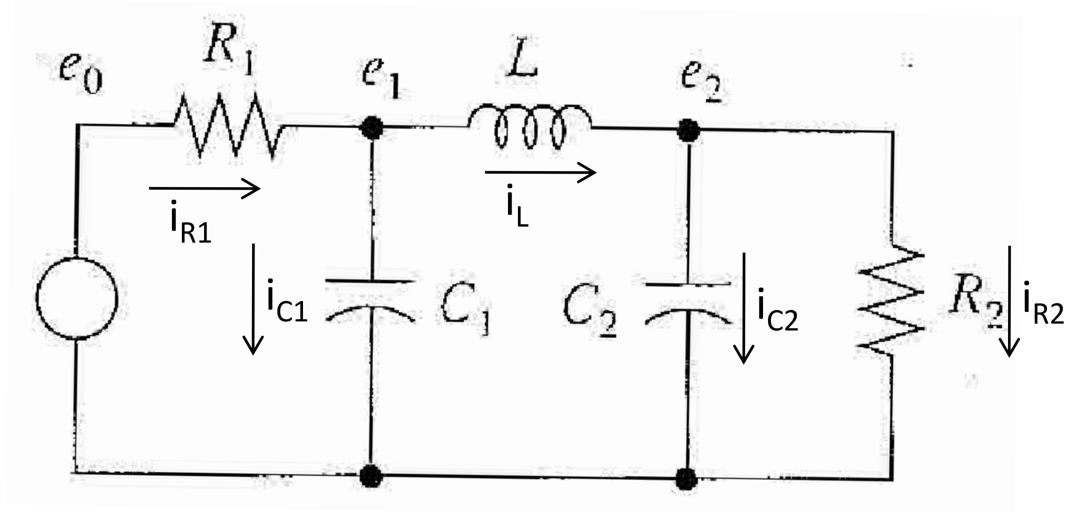
Order of differential equations will be equal to number of capacitors and inductors that are not connected in trivial manner (e.g., two capacitors in series/parallel with no R or L between them)

Example 5: Pair-Share: RLC Circuit



- For the circuit shown above, write all modeling equations and derive the transfer function e_2/e_0. All initial conditions are zero.
- Derive the state-space representation of the system

Example 5: Pair-Share: RLC Circuit



Component equations

$$i_{R_1} = \frac{e_0 - e_1}{R_1}$$

$$i_{C_1} = C_1\, De_1 \quad \text{with } e_1(0)$$

$$i_L = \frac{e_1 - e_2}{L\,D} \quad \text{with } i_L(0)$$

$$i_{C_2} = C_2\, De_2 \quad \text{with } e_2(0)$$

$$i_{R_2} = \frac{e_2}{R_2}$$

Example 5: Pair-Share: RLC Circuit

Node equations $\quad i_{R_1} = i_{C_1} + i_L, \quad i_L = i_{C_2} + i_{R_2}$

Substitute component equations into node equations.

$$\frac{e_0 - e_1}{R_1} = C_1\, De_1 + \frac{e_1 - e_2}{LD} \qquad \text{or} \qquad \left[L\, C_1\, D^2 + \frac{L}{R_1} D + 1 \right] e_1 = \frac{LD}{R_1} e_0 + e_2$$

$$\frac{e_1 - e_2}{LD} = C_2\, De_2 + \frac{e_2}{R_2} \qquad \text{or} \qquad \left[L\, C_2\, D^2 + \frac{L}{R_2} D + 1 \right] e_2 = e_1$$

Combining to solve for e_2 as a function of e_0:

$$\left[L\, C_1\, D^2 + \frac{L}{R_1} D + 1 \right] \left[L\, C_2\, D^2 + \frac{L}{R_2} D + 1 \right] e_2 = \frac{LD}{R_1} e_0 + e_2$$

Normalizing,

$$\frac{e_2}{e_0} = \frac{\dfrac{1}{(1 + R_1/R_2)}}{\dfrac{R_1 C_1 L C_2}{(1 + R_1/R_2)} D^3 + \dfrac{\left(LC_2 + \dfrac{R_1}{R_2} LC_1 \right)}{(1 + R_1/R_2)} D^2 + \dfrac{\left(R_1 C_1 + R_2 C_2 + \dfrac{L}{R_2} \right)}{(1 + R_1/R_2)} D + 1}$$

Example 5: Pair-Share: RLC Circuit

Node equations $i_{R_1} = i_{C_1} + i_L, \quad i_L = i_{C_2} + i_{R_2}$

Component equations

$$i_{R_1} = \frac{e_0 - e_1}{R_1}$$

$$i_{C_1} = C_1\, De_1 \quad \text{with } e_1(0)$$

$$i_L = \frac{e_1 - e_2}{L\, D} \quad \text{with } i_L(0)$$

$$i_{C_2} = C_2\, De_2 \quad \text{with } e_2(0)$$

$$i_{R_2} = \frac{e_2}{R_2}$$

Defining state variables

$$u_1 = e_0, \quad x_1 = e_1, \quad x_2 = i_L, \quad x_3 = e_2$$

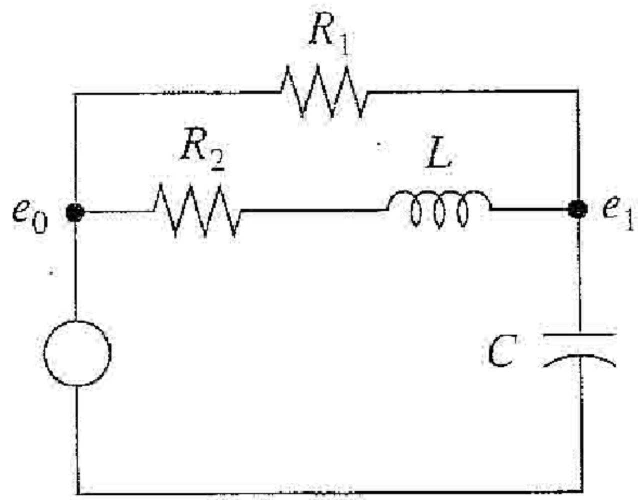
State-space representation

$$\dot{x}_1 = -\frac{1}{R_1\, C_1} x_1 - \frac{1}{C_1} x_2 + \frac{1}{R_1\, C_1} u_1$$

$$\dot{x}_2 = \frac{1}{L} x_1 - \frac{1}{L} x_3$$

$$\dot{x}_3 = -\frac{1}{C_2} x_2 - \frac{1}{R_2\, C_2} x_3$$

Example 6: RLC Circuit With Parallel Bypass Resistor



- For the circuit shown above, write all modeling equations and derive a differential equation for e_1 as a function of e_0. Express required initial conditions of this second-order differential equations in terms of known initial conditions $e_1(0)$ and $i_L(0)$.
- Derive the state-space representation of the system using variables e_1 and i_L

Example 6: RLC Circuit With Parallel Bypass Resistor

Component equations

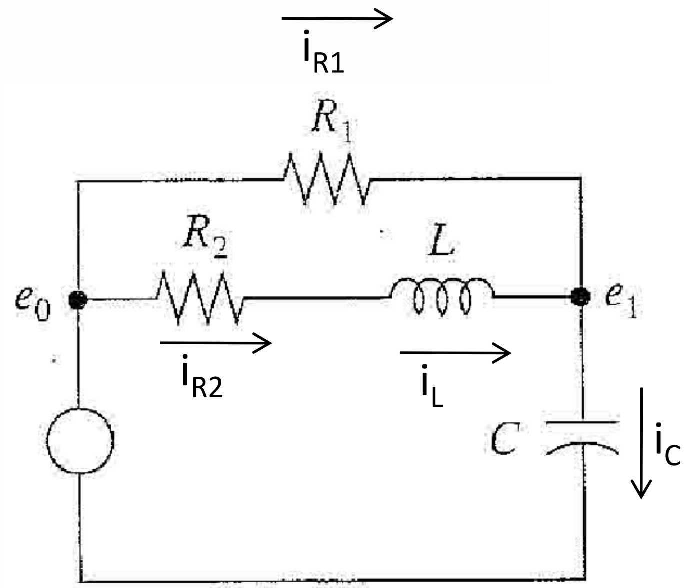
$$i_{R_1} = \frac{e_0 - e_1}{R_1}$$

$$i_L = \frac{e_0 - e_1}{R_2 + L\,D} \quad \text{with } i_L(0)$$

$$i_C = C\,De_1 \quad \text{with } e_1(0)$$

Node equation

$$i_{R_1} + i_L = i_C$$



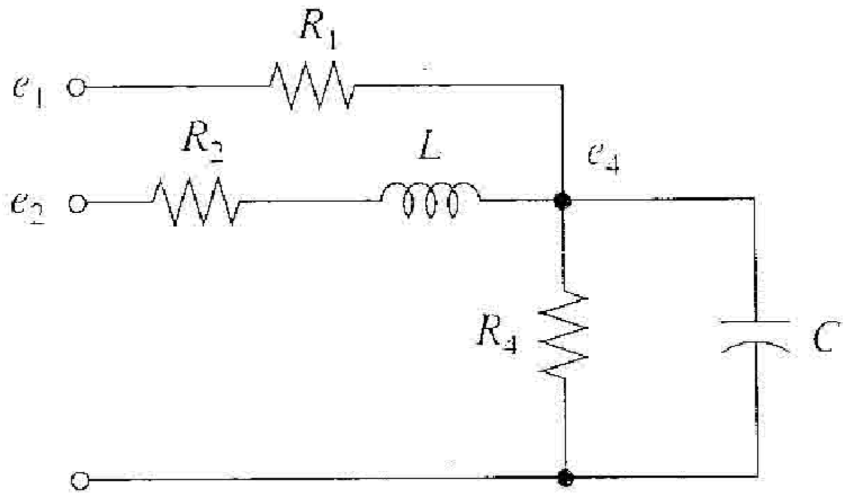
Substituting component equations into node equation and simplifying

$$\left[\frac{L\,C}{\left(1+\frac{R_2}{R_1}\right)}D^2 + \frac{\left(R_2\,C + \frac{L}{R_1}\right)}{\left(1+\frac{R_2}{R_1}\right)}D + 1\right]e_1 = \left[\frac{\frac{L}{R_1}}{\left(1+\frac{R_2}{R_1}\right)}D + 1\right]e_0$$

The initial conditions (converted from initial conditions of voltages and currents to e_1 and $\dot{e}_1$,

$$e_1(0) = \text{known}, \quad \dot{e}_1 = \frac{1}{C}i_L(0) + \frac{e_0(0) - e_1(0)}{R_1\,C}$$

Example 7: Pair-Share: RLC Circuit With Two Voltage Inputs



- For the circuit shown above, write all modeling equations and derive a transfer function relating e_4 as a function of inputs e_1 and e_2.
- Derive a state-space representation of the system using two state variables and two inputs.
- What are the initial conditions of the state variables?

Example 7: Pair-Share: RLC Circuit With Two Voltage Inputs

Component equations

$$i_{R_1} = \frac{e_1 - e_4}{R_1}$$

$$i_{R_2} = \frac{e_2 - e_3}{R_2}$$

$$i_L = \frac{e_3 - e_4}{L\,D} \qquad \text{with} \quad i_L(0)$$

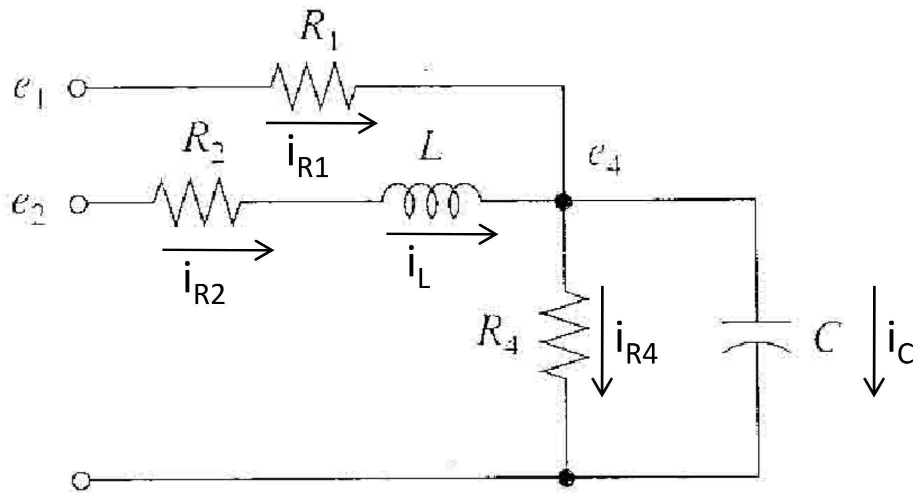
$$i_{R_4} = \frac{e_4}{R_4}$$

$$i_C = C\,De_4 \qquad \text{with} \quad e_4(0)$$

Node equations

$$i_{R_2} = i_L$$

$$i_{R_1} + i_L = i_{R_4} + i_C$$



Example 7: Pair-Share: RLC Circuit With Two Voltage Inputs

Substitute component equations into node equations.

$$\frac{e_2 - e_3}{R_2} = \frac{e_3 - e_4}{LD} \qquad \text{or} \qquad \left[\frac{L}{R_2} D + 1\right] e_3 = \frac{LD}{R_2} e_2 + e_4$$

$$\frac{e_1 - e_4}{R_1} + \frac{e_3 - e_4}{L\,D} = \frac{e_4}{R_4} + C\,De_4 \qquad \text{or} \qquad \left[L\,C\,D^2 + \frac{L}{R_4} D + \frac{L}{R_1} D + 1\right] e_4 = \frac{L\,D}{R_1} e_1 + e_3$$

Reduce to get e_4 as a function of e_1 and e_2.

$$e_4 = \frac{\frac{R_2}{R_1}\left(\frac{L}{R_2} D + 1\right) e_1 + e_2}{L\,C\,D^2 + \left(R_2 C + \frac{L}{R_1} + \frac{L}{R_4}\right) D + \left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_4}\right)}$$

Example 7: Pair-Share: RLC Circuit With Two Voltage Inputs

State-space definitions

$$u_1 = e_1$$

$$u_2 = e_2$$

$$x_1 = i_L \qquad \text{thus,} \quad \dot{x}_1 = Di_L = \frac{e_3 - e_4}{L} = \frac{1}{L}e_3 - \frac{1}{L}x_2$$

$$x_2 = e_4 \qquad \text{thus,} \quad \dot{x}_2 = De_4 = \frac{1}{C}i_C$$

We need to eliminate e_3, and i_C. From the R_2 component equation and the first node equation,

$$e_3 = e_2 - R_2\, i_{R_2} = e_2 - R_2\, i_L = u_2 - R_2\, x_1$$

Component equations

$$i_{R_1} = \frac{e_1 - e_4}{R_1}$$

$$i_{R_2} = \frac{e_2 - e_3}{R_2}$$

$$i_L = \frac{e_3 - e_4}{L\,D} \qquad \text{with} \quad i_L(0)$$

$$i_{R_4} = \frac{e_4}{R_4}$$

$$i_C = C\,De_4 \qquad \text{with} \quad e_4(0)$$

Node equations

$$i_{R_2} = i_L$$

$$i_{R_1} + i_L = i_{R_4} + i_C$$

Example 7: Pair-Share: RLC Circuit With Two Voltage Inputs

From the second node equation and the R_4 component equation,

$$i_C = i_L + i_{R_1} - i_{R_4} = i_L + \frac{e_1 - e_4}{R_1} - \frac{e_4}{R_4} = x_1 - \left(\frac{1}{R_1} + \frac{1}{R_4}\right)x_2 + \frac{1}{R_1}u_1$$

State-space representation

$$\dot{x}_1 = -\frac{R_2}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u_2$$

$$\dot{x}_2 = \frac{1}{C}x_1 - \left(\frac{1}{R_1\,C} + \frac{1}{R_4\,C}\right)x_2 + \frac{1}{R_1\,C}u_1$$

With the initial conditions

$$x_1(0) = i_L(0)$$

$$x_2(0) = e_4(0)$$

Active Circuit Analysis

Electrical System

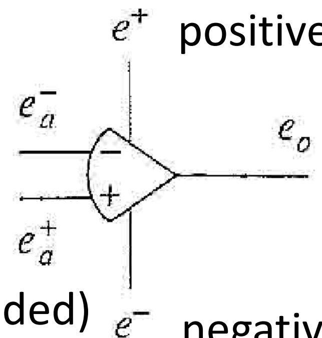
- Composed of resistors, capacitors, inductors, transistors, amplifiers, power supplies
  - Passive circuits: respond to applied voltage or current and do not have any amplifiers
  - Active circuits: made of transistors and/or amplifiers, require active power source to work
- Basic quantities
  - Charge q [coulomb] = 6.24×10^{18} electrons
  - Current i [ampere] = dq/dt
  - Voltage e [Volt] = dw/dq
  - Energy or Work w [joule]
  - Power p [watt] = e x i = dw/dt

Operational Amplifier

- Op-amp: integrated circuit that amplifies voltage

positive power supply

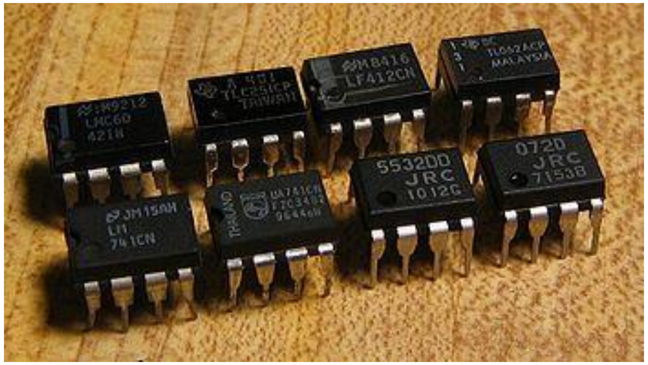
inverting input



output

non-inverting input
(reference, usu. grounded)

negative power supply



- Key properties
  - High gain (> 10^6 volt/volt) -> ideal computation device
  - Low output impedance (< 100 ohms) -> output voltage does not vary with output current, so amplifier drives loads as ideal voltage source
  - High input impedance (10^6 ohms) and low input voltage -> no current is required by amplifier
  - Idealizations: zero noise, infinite bandwidth

Operational Amplifier

- Component equations:

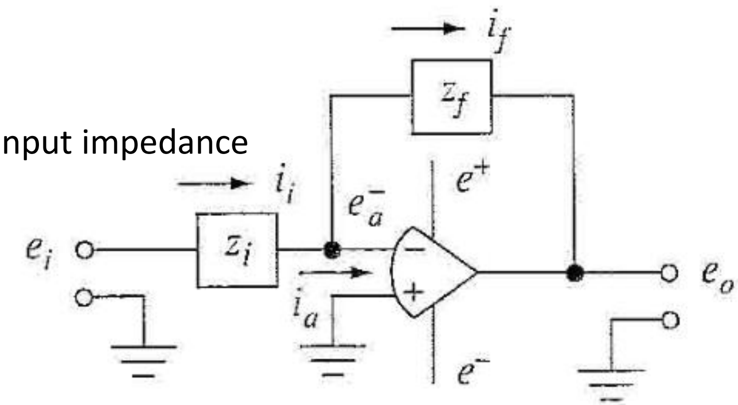
$$i_i = \frac{e_i - e_a^-}{Z_i}$$

$$i_f = \frac{e_a^- - e_o}{Z_f}$$

$$e_o = -Ge_a^-$$

Z_f: feedback impedance

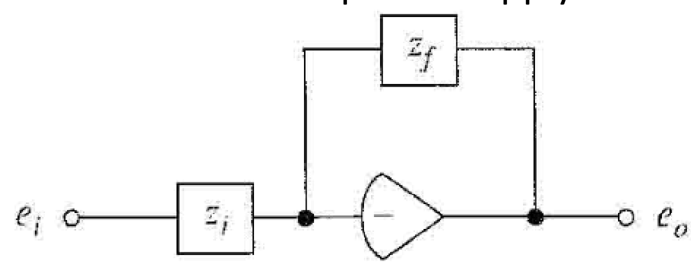
Z_i: input impedance



- Node equation:

$$i_i - i_f = i_a \approx 0$$

Input is grounded and differential power supply is used



- Substitute component eqs. into node eq:

$$e_o = \frac{-\frac{Z_f}{Z_i} e_i}{1 + \frac{1 + \frac{Z_f}{Z_i}}{G}} = -\frac{Z_f}{Z_i} e_i$$

Z_f/Z_i is small compared to G

TABLE 4.1 Op-Amp Circuits.

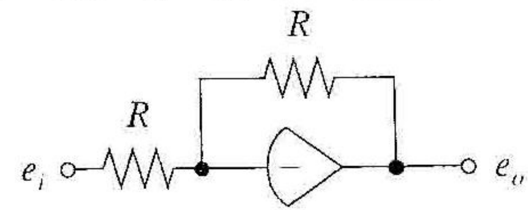
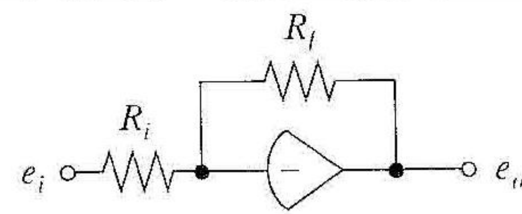
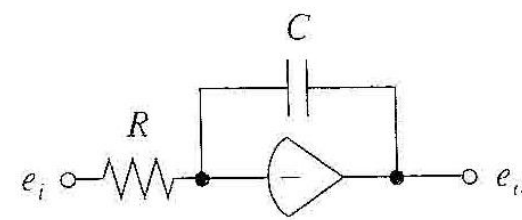
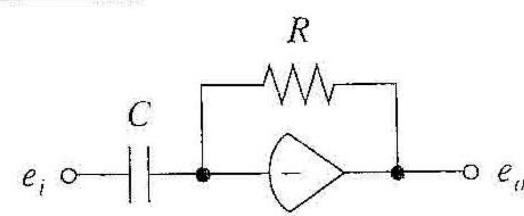
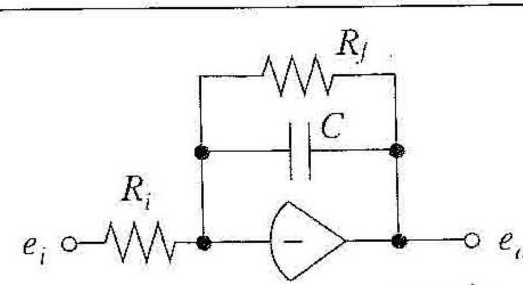
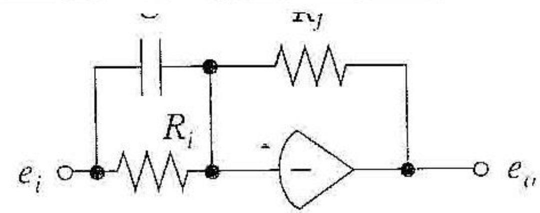
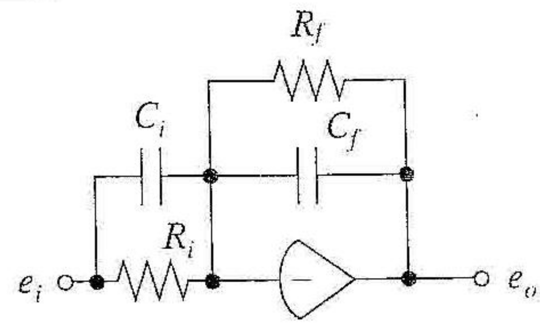
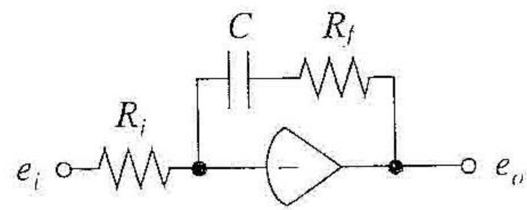
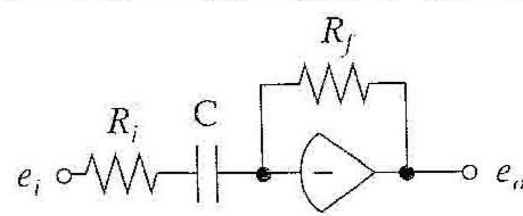
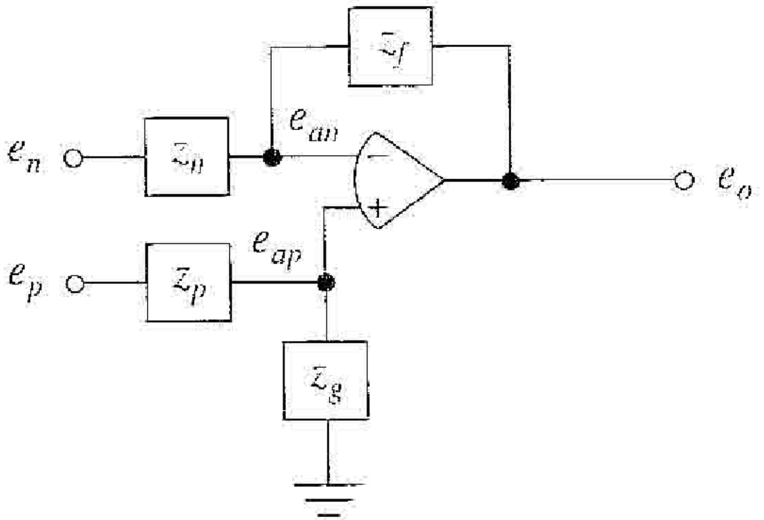
Description	Transfer Function	Circuit
Sign Changer	$e_o = -e_i$	
Amplifier	$e_o = -\frac{R_f}{R_i} e_i$	
Integrator	$e_o = \frac{-e_i}{\tau D}$ $\tau = RC$	
Differentiator	$e_o = -\tau D e_i$ $\tau = RC$	
Lag	$e_o = \frac{-\frac{R_f}{R_i} e_i}{(\tau D + 1)}$ $\tau = R_f C$	

TABLE 4.1 Op-Amp Circuits.

Description	Transfer Function	Circuit
Lead	$e_o = -\frac{R_f}{R_i}(\tau D + 1)$ $\tau = R_i C$	
Lead-Lag or Lag-Lead	$e_o = -\frac{R_f}{R_i}\frac{(\tau_i D + 1)e_i}{(\tau_f D + 1)}$ $\tau_i = R_i C_i$ $\tau_f = R_f C_f$	
Bandwidth-Limited Integrator	$e_o = \frac{-(\tau_f D + 1)\, e_i}{\tau_i D}$ $\tau_f = R_f C$ $\tau_i = R_i C$	
Bandwidth-Limited Differentiator	$e_o = \frac{-\tau_f D e_i}{(\tau_i D + 1)}$ $\tau_f = R_f C$ $\tau_i = R_i C$	

Example 8: Op-Amp Circuit



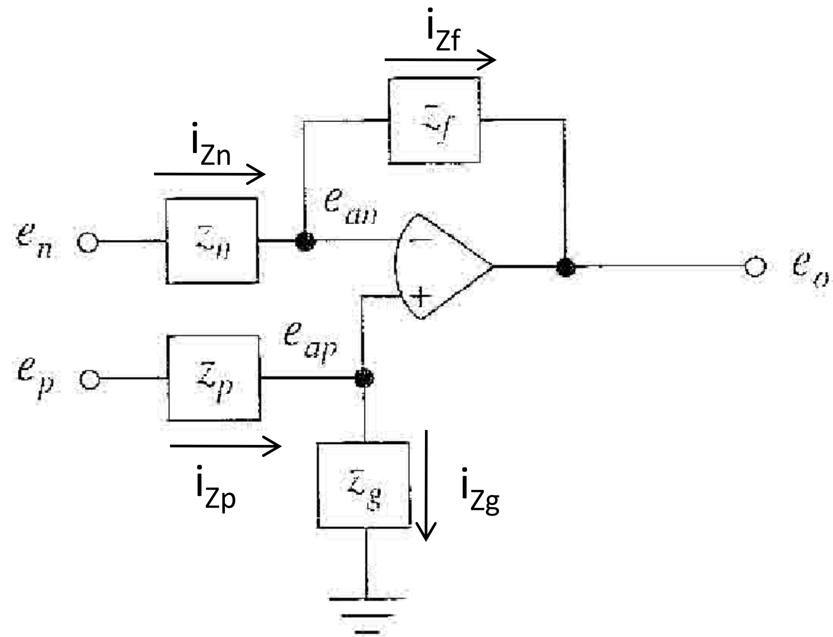
- Above is a an op-amp circuit with impedances on the plus and minus inputs, derive the output equation e_0 as a function of e_n and e_p. The amplifier has characteristic $e_0=G(e_{ap}-e_{an})$, where G >> 1.
- Show that if all impedances are resistive and equal to R, then $e_0=e_p-e_n$.

Example 8: Op-Amp Circuit

On the negative input, the component equations are

$$i_{Z_n} = \frac{e_n - e_{an}}{Z_n}$$

$$i_{Z_f} = \frac{e_o - e_{an}}{Z_f}$$



On the positive input, the component equations are

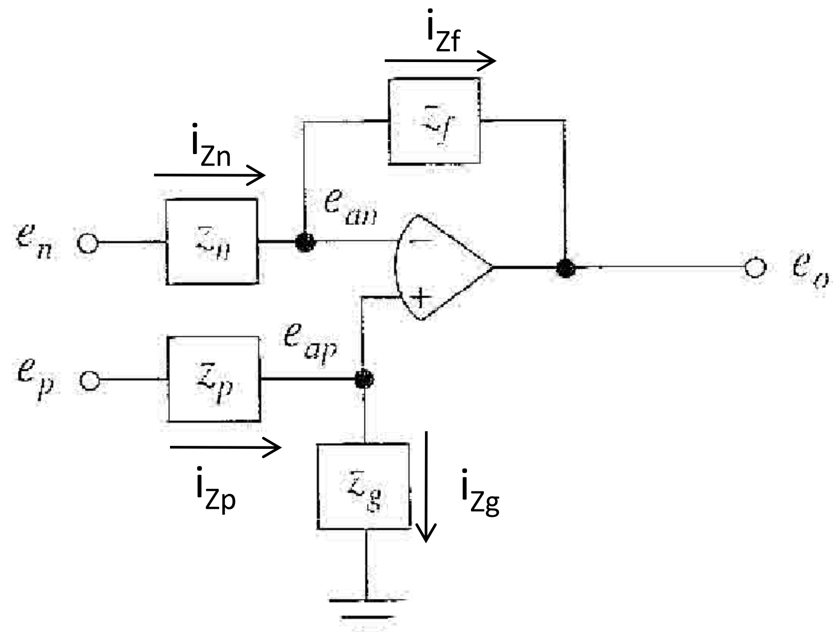
$$i_{Z_p} = \frac{e_p - e_{ap}}{Z_p}$$

$$i_{Z_g} = \frac{e_{ap}}{Z_g}$$

The amplifier equation

$$e_o = G(e_{ap} - e_{an})$$

Example 8: Op-Amp Circuit



Node equations

$$i_{Z_n} + i_{Z_f} = i_{an} \approx 0$$

$$i_{Z_p} - i_{Z_g} = i_{ap} \approx 0$$

Substituting component equations into node equations

$$\frac{e_n - e_{an}}{Z_n} + \frac{e_o - e_{an}}{Z_f} = 0 \qquad \text{or} \qquad \frac{e_n}{Z_n} + \frac{e_o}{Z_f} = \left(\frac{1}{Z_n} + \frac{1}{Z_f}\right) e_{an}$$

$$\frac{e_p - e_{ap}}{Z_p} - \frac{e_{ap}}{Z_g} = 0 \qquad \text{or} \qquad \frac{e_p}{Z_p} = \left(\frac{1}{Z_p} + \frac{1}{Z_g}\right) e_{ap}$$

Example 8: Op-Amp Circuit

From the amplifier equation and the above

$$\frac{e_o}{G} = e_{ap} - e_{an} = \left[\frac{\dfrac{e_p}{Z_p}}{\dfrac{1}{Z_p}+\dfrac{1}{Z_g}}\right] - \left[\frac{\dfrac{e_n}{Z_n}+\dfrac{e_o}{Z_f}}{\dfrac{1}{Z_n}+\dfrac{1}{Z_f}}\right] = \left[\frac{e_p}{1+\dfrac{Z_p}{Z_g}}\right] - \left[\frac{e_n + \dfrac{Z_n}{Z_f}e_o}{1+\dfrac{Z_n}{Z_f}}\right]$$

Thus the output voltage is

$$\left[\frac{1}{G} + \frac{\dfrac{Z_n}{Z_f}}{1+\dfrac{Z_n}{Z_f}}\right] e_o = \left[\frac{e_p}{1+\dfrac{Z_p}{Z_g}}\right] - \left[\frac{e_n}{1+\dfrac{Z_n}{Z_f}}\right]$$

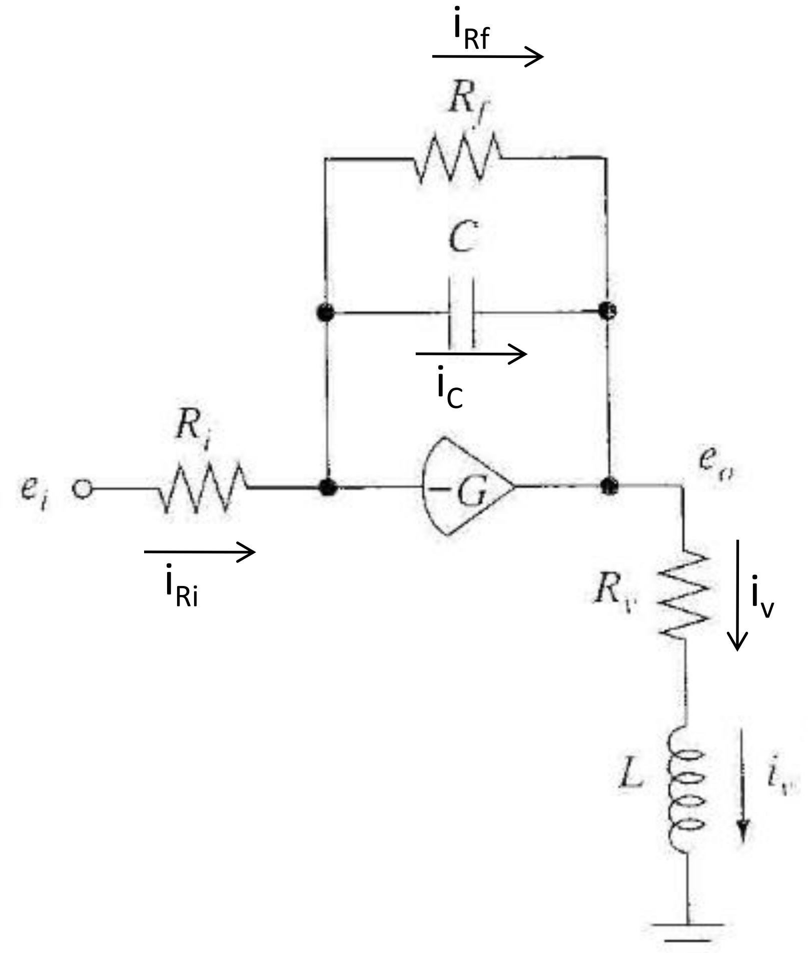
Since $G >> 1 + \dfrac{Z_f}{Z_n}$, then $\dfrac{1}{G} \approx 0$, and the above equation reduces to the following.

$$e_o = \left[\frac{1+\dfrac{Z_n}{Z_f}}{1+\dfrac{Z_p}{Z_g}}\right] e_p - \frac{Z_f}{Z_n} e_n$$

If all of the impedances are resistive and equal to R, then this reduces to

$$e_o = e_p - e_n$$

Example 9: Pair-Share: Op-Amp Circuit



- Above is a an op-amp circuit used to drive an electromagnetic coil on a servo valve. Write all the modeling equations and derive the transfer function for i_v as a function of input voltage e_i.
- Derive a state-space representation for the system.

Example 9: Pair-Share: Op-Amp Circuit

We can state the op-amp equation as follows

$$e_o = \frac{-\dfrac{R_f}{R_i}}{R_f C\, D + 1} e_i \qquad \text{with} \quad e_o(0)$$

Since the op-amp has a very low output impedance and can thus be treated as a voltage source, we can consider that the coil is being driven by a voltage source, e_o. Therefore, the current in the valve coil is

$$i_v = \frac{e_o}{R_v + L\, D} \qquad \text{with} \quad i_v(0)$$

Thus, the transfer equation is

$$i_v = \frac{-\dfrac{R_f}{R_i}\dfrac{1}{R_v}}{\left(R_f C\, D + 1\right)\left(\dfrac{L}{R_v} D + 1\right)} e_i$$

Example 9: Pair-Share: Op-Amp Circuit

Component equations

$$e_o = \frac{-\frac{R_f}{R_i}}{R_f\,C\,D+1}\,e_i \quad \text{with} \quad e_o(0)$$

$$i_v = \frac{e_o}{R_v + L\,D} \quad \text{with} \quad i_v(0)$$

Definition of state variables

$$u_1 = e_i$$

$$x_1 = e_o$$

$$x_2 = i_v$$

State-space representation

$$\dot{x}_1 = -\frac{1}{R_f\,C}x_1 - \frac{1}{R_i\,C}u_1$$

$$\dot{x}_2 = \frac{1}{L}x_1 - \frac{R_v}{L}x_2$$

Initial conditions

$$x_1(0) = e_o(0)$$

$$x_2(0) = i_v(0)$$

Example 10: Full-Bridge Strain Gauge Circuit

A full-bridge strain gauge circuit is used to measure the force applied to a bar by measuring the strain in the small steel bar. The strain gauge resistance elements are 350 Ω. The bridge is driven with a 5 volt power source. As the strain is applied, one of the bridge resistors changes resistance according to the relationship $R_2 = 350 + cF$, where F is the applied force and the coefficient c works out to be 0.7 Ω/N (taking into consideration the size and stiffness of the bar and the sensitivity of the strain gauge element). From this basic bridge circuit, a differential voltage Δe_b, can be expressed as a function of the force to be measured. Since this voltage differential is very small, it is desired to use an op-amp circuit to amplify the signal into the 10 volt range. In addition, because there are some unwanted vibrations in the bar, it is required to filter the measured force signal to eliminate the high-frequency vibrations, which are expected to be a few hundred hertz. A first-order system will suffice.

The exact requirements are that a ±20 N force should produce a ±10 volt output signal from the op-amp circuit. The overall circuit should have a dynamic time constant of 1.5 milliseconds. *Draw* a complete circuit diagram of this system, including the strain gauge bridge circuit with amplifier. *Derive* a complete mathematical model of the system from force input to voltage output. *Select* values for all resistors and capacitors.

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The bridge circuit equation for a full bridge with one active element is

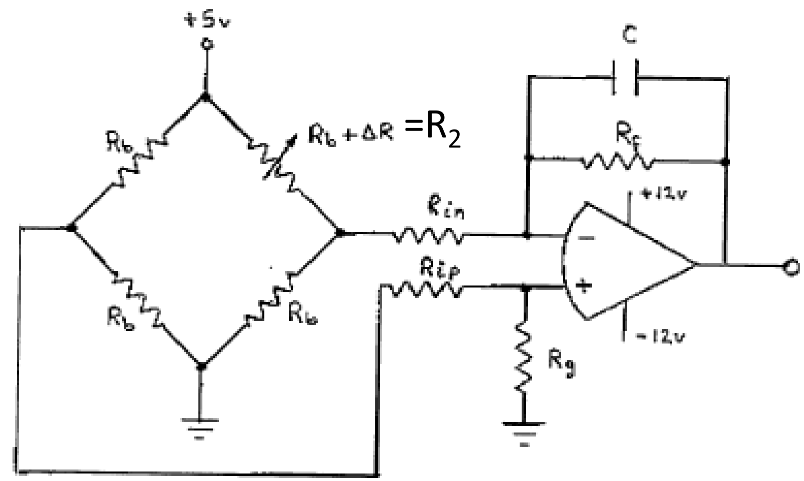
$$\Delta e_b = \frac{\delta R}{2\,R}\,e^+$$

In this circuit, the variation in resistance is related to the force.

$$R_2 = 350\,\Omega + c\,F \qquad \text{thus} \qquad \delta R = c\,F$$

This signal is amplified with an op-amp having a low-pass filter. The transfer function is given below.

$$e_o = \frac{-\dfrac{R_f}{R_i}}{\tau D + 1}\,\Delta e_b \qquad \text{where} \quad \tau = R_f\,C_f$$



The overall circuit equation is

$$e_o = \frac{-\dfrac{R_f}{R_i}\left(\dfrac{\delta R}{2\,R}\right)}{\tau D + 1}\,e^+ = \frac{-\dfrac{R_f}{R_i}\left(\dfrac{c\,F}{2\,R}\right)}{\tau D + 1}\,e^+$$

Thus the static gain of the system is $\quad G_s = -\dfrac{R_f}{R_i}\left(\dfrac{c\,e^+}{2\,R}\right)$

Example 10: Full-Bridge Strain Gauge Circuit

The desired static gain is 10 volts / 20 N, using the stated values we find

$$G_s = -\frac{R_f}{R_{in}}\left(\frac{0.7\,\dfrac{\text{ohm}}{\text{N}}\,5\text{ volt}}{2\times 350\text{ ohm}}\right) = \frac{10\text{ volt}}{20\text{ N}}$$

Thus, the static gain should be

$$\frac{R_f}{R_{in}} = \frac{\dfrac{10\text{ volt}}{20\text{ N}}}{\left(\dfrac{0.7\,\dfrac{\text{ohm}}{\text{N}}\,5\text{ volt}}{2\times 350\text{ ohm}}\right)} = 100$$

The dynamic characteristic of the system requires that $\tau = 0.0015$ s. Using a value of 10 k Ω for R_{in} requires that R_f be 1 MΩ for the static gain. The required capacitance is

$$R_f\,C = 0.0015\text{ s} \qquad \text{thus} \qquad C = \frac{0.0015\text{ s}}{1{,}000{,}000\text{ ohm}} = 0.0015\,\mu\text{f}$$

It might have been better to use two op-amps, one as a buffer amplifier, and the second as the filter. In this way, the feedback resistance of 1 MΩ (which is a little high) could have been avoided.

Example 11: Pair-Share: Audio Amplifier Circuit w/ Light Bulb

A flashing light is to be placed on the output of an audio amplifier to show the intensity of the sound coming from the speaker. You must select an impedance for this lightbulb that will not degrade the voltage going to the speaker. Intuitively, if the impedance is very large relative to the output impedance of the amplifier, then there will be no degradation; however, if the light resistance starts approaching the impedance of the speaker, then a considerable amount of power will be going into the light, and the speaker and the sound will be degraded. This is clearly undesirable.

Derive an expression for the voltage at the speaker in the undisturbed circuit (Figure P4.29(a)) and the circuit loaded with the light (Figure P4.29(b)). The output impedance R_o of the amplifier is 8 Ω, and the impedance of the speaker, $R_{speaker}$, is also 8 Ω. At *what* value of light impedance R_{light} is there a 1% degradation in the voltage going to the speaker (i.e., at what value of light impedance will the gain be 0.99 of the undisturbed gain)?

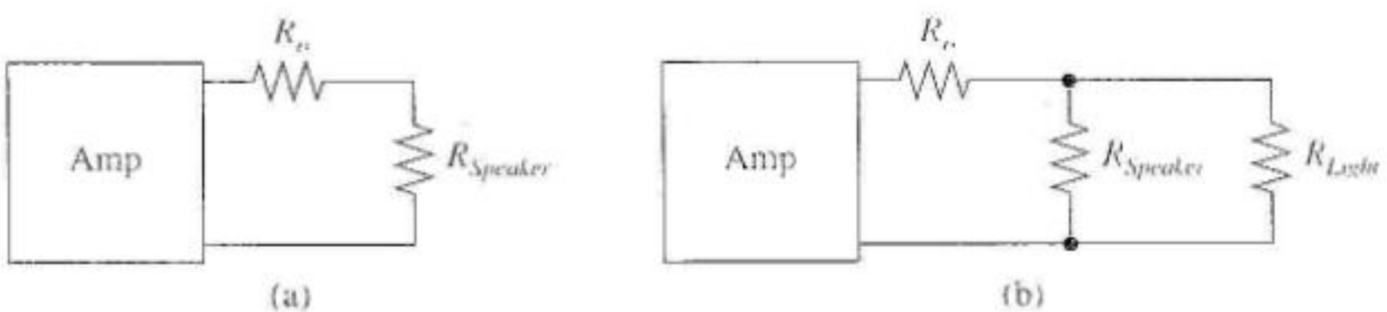


Figure P4.29 Audio amplifier circuit with light bulb. (a) Normal audio circuit of amplifier and speaker. (b) Modified circuit with lightbulb.

Example 11: Pair-Share: Audio Amplifier Circuit w/ Light Bulb

The transfer function for the voltage output (voltage to the speaker) relative to the internal voltage of the undisturbed system can be stated from the voltage divider equation.

$$e_o = \frac{R_{speaker}}{R_o + R_{speaker}} e_a = \frac{1}{1 + \frac{R_o}{R_{speaker}}} e_a$$

The voltage to the speaker with the light attached is given by

$$e_o = \frac{\left(\frac{R_{speaker} R_{light}}{R_{speaker} + R_{light}}\right)}{R_o + \left(\frac{R_{speaker} R_{light}}{R_{speaker} + R_{light}}\right)} e_a = \frac{1}{1 + \frac{R_o}{R_{speaker}}\left(1 + \frac{R_{speaker}}{R_{light}}\right)} e_a$$

For the gain of the system with the light to be equal to 0.99 of the gain of the original system, we must have

$$\frac{1}{1 + \frac{R_o}{R_{speaker}}\left(1 + \frac{R_{speaker}}{R_{light}}\right)} = \frac{0.99}{1 + \frac{R_o}{R_{speaker}}}$$

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Thus,

$$\frac{R_{light}}{R_{speaker}} = \frac{1}{\left[\frac{1}{0.99}\left(1+\frac{R_{speaker}}{R_o}\right)-\frac{R_{speaker}}{R_o}-1\right]}$$

Using numbers given for the impedances,

$$\frac{R_{light}}{R_{speaker}} = \frac{1}{\left[\frac{1}{0.99}\left(1+\frac{8}{8}\right)-\frac{8}{8}-1\right]} = 49.5$$

Therefore, the light resistance can be 49.5 times the speaker impedance, or 396 Ω. At 12 volts, this bulb would require 0.36 watts.