

COMP 222 Quick Quiz #3: Hamming Codes Solutions

Problem 1 Given the 8-bit data word 0x57, compute the 12-bit code word.

Expand data word to binary and place in table

0	1	0	1	0	1	1	1
8	7	6	5	4	3	2	1

Calculate check bits (xor of indicated data bits)

Bit 1	12457	Xor(1 1 0 1 1) = 0
Bit 2	13467	Xor(1 1 0 0 1) = 1
Bit 3	2348	Xor(1 1 0 0) = 0
Bit 4	5678	Xor(1 0 1 0) = 0

Reposition original data bits and calculated check bits into 12-bit code word.

0 (D ₈)	1 (D ₇)	0 (D ₆)	1 (D ₅)	0 (C ₄)	0 (D ₄)	1 (D ₃)	1 (D ₂)	0 (C ₃)	1 (D ₁)	1 (C ₂)	0 (C ₁)
12	11	10	9	8	7	6	5	4	3	2	1

Code word in hex: _0101 0011 0110 = **536**₁₆_

Problem 2 Given the 8-bit data word 0xAB, compute the 12-bit code word.

Expand data word to binary and place in table (0xAB = 1010 1011)

1	0	1	0	1	0	1	1
8	7	6	5	4	3	2	1

Calculate check bits (xor of indicated data bits)

Bit 1	12457	Xor(1 1 1 0 0) = 1
Bit 2	13467	Xor(1 0 1 1 0) = 1
Bit 3	2348	Xor(1 0 1 1) = 1
Bit 4	5678	Xor(0 1 0 1) = 0

Reposition original data bits and calculated check bits into 12-bit code word.

1 (D ₈)	0 (D ₇)	1 (D ₆)	0 (D ₅)	0 (C ₄)	1 (D ₄)	0 (D ₃)	1 (D ₂)	1 (C ₃)	1 (D ₁)	1 (C ₂)	1 (C ₁)
12	11	10	9	8	7	6	5	4	3	2	1

Code word in hex: _1010 0101 1111 = **A5F**₁₆_

Problem 3 Given 12-bit code word 0x7C1, extract the data bits, determine bit error if any, and if so, correct it.

Expand 12-bit code word into binary. 0x7C1 = 0111 1100 0001

0 (D ₈)	1 (D ₇)	1 (D ₆)	1 (D ₅)	1 (C ₄)	1 (D ₄)	0 (D ₃)	0 (D ₂)	0 (C ₃)	0 (D ₁)	0 (C ₂)	1 (C ₁)
12	11	10	9	8	7	6	5	4	3	2	1

Extract data bits from code word into table.

0	1	1	1	1	0	0	0
8	7	6	5	4	3	2	1

Recalculate check bits (xor of indicated data bits)

Bit 1	12457	Xor(0 0 1 1 1) = 1
Bit 2	13467	Xor(0 0 1 1 1) = 1
Bit 3	2348	Xor(0 0 1 0) = 1
Bit 4	5678	Xor(1 1 1 0) = 1

Compare stored check bits with recalculated check bits

	Bit 4	Bit 3	Bit 2	Bit 1
Original	1	0	0	1
Recalculated	1	1	1	1
Xor	0	1	1	0

Code Word Bit In Error: 0110 = 6 = Data Bit In Error 3

Corrected Data Word in Hex: 0111 1000 = 0111 1100 = **7C**₁₆
(same as extracted data word if no error)

Problem 4 Given 12-bit code word 0x9DD, extract the data bits, determine bit error if any, and if so, correct it.

Expand 12-bit code word into binary.

1	0	0	1	1	1	0	1	1	1	0	1
12	11	10	9	8	7	6	5	4	3	2	1

Extract data bits from code word into table.

1	0	0	1	1	0	1	1
8	7	6	5	4	3	2	1

Recalculate check bits (xor of indicated data bits)

Bit 1	12457	Xor(1 1 1 1 0) = 0
Bit 2	13467	Xor(1 0 1 0 0) = 0
Bit 3	2348	Xor(1 0 1 1) = 1
Bit 4	5678	Xor(1 0 0 1) = 0

Compare stored check bits with recalculated check bits

	Bit 4	Bit 3	Bit 2	Bit 1
Original	1	1	0	1
Recalculated	0	1	0	0
Xor	1	0	0	1

Code Word Bit In Error: 1001 = 9 = Data Bit In Error 5

Corrected Data Word in Hex: 1001 1011 = 1000 1011 = **8B**₁₆
(same as extracted data word if no error)