

# A New Methodology for Probabilistic Short-Circuit Evaluation With Applications in Power Quality Analysis

Ulisses A. Bordalo, Anselmo B. Rodrigues, and Maria G. Da Silva, *Member, IEEE*

**Abstract**—The main aim of this paper is to propose a probabilistic short-circuit approach to generate the probability distributions of the System Average RMS Variation Index (SARFI) index. The proposed methodology is based on the combination of the admittance summation method in phase coordinates with the Monte Carlo method. This new approach has been tested in a feeder belonging to an electricity distribution company in the Northeast of Brazil to generate the probability distributions of the SARFI index. The tests results demonstrated that the proposed model is a powerful tool to stochastic prediction of voltage sags.

**Index Terms**—Monte Carlo methods, power distribution planning, power quality, short-circuit currents.

## NOMENCLATURE

$n$	System's bus number.
$Y_{LK}, J_{LK}$	Admittance matrix ( $3 \times 3$ ) and the current injection vector ( $3 \times 1$ ) associated to the bus $k$ , respectively. The admittance matrix $Y_{LK}$ equals to the summatory of the loads admittance shunts susceptance and faults connected to the bus $k$ .
$Y_{E_k}, J_{E_k}$	Three-phase equivalent admittance matrix ( $3 \times 3$ ) and the currents injection vector of equivalent phases ( $3 \times 1$ ) associated to the bus $k$ , respectively.
$I$	Identity matrix ( $3 \times 3$ ).
$ZS_{ik}$	Primitive impedance matrix ( $3 \times 3$ ) of branch $i - k$ .
$Zp_{KL}$	Primitive impedance matrix ( $3 \times 3$ ) of branch $k - l$ .
$\bar{Z}_{PKL}$	Normalized primitive impedance matrix ( $3 \times 3$ ) given in $\Omega/\text{mile}$ .
$V_k$	Phase voltages vector ( $3 \times 1$ ) for node $k$ .
$d$	Length of the branch $k - l$ .
$x$	Distance between the fault node and initial node of the line that suffered the fault.
$Y_f$	Fault admittance matrix ( $3 \times 3$ ).

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## I. INTRODUCTION

NOWADAYS, electricity utilities receive many complaints due to power quality problems associated with interruptions and voltage sags. The main reason for this is the increasing use of new equipment in all sectors (residential, commercial, and industrial), which are more susceptible to voltage variations [1]. Consequently, it is very important to study power quality problems associated with voltage sags. The purpose of these studies is to evaluate quantitative indexes for voltage sags and to analyze their characteristics. Electricity utilities use the results of these studies to evaluate the system's performance level against sags and also to study alternative configurations for the mitigation of their effects.

Currently, quantitative voltage sags assessment is carried out using two techniques: past performance measuring (monitoring) and future performance prediction (probabilistic technique) [2]. The technique of past performance measuring of voltage sags applied to distribution networks is the process in which data are stored and analyzed. This analysis provides important information about sags. When there are system changes, such as novel operational procedures or installation of new protection devices, the collected monitoring data cannot represent the actual system performance. Since faults are the principal source of voltage sags and they occur randomly, voltage sags monitoring for a single area requires a long time to be completed.

In future performance predictions, two probabilistic techniques are used: the analytical technique and the stochastic simulation [2]. The analytical technique represents the system by means of mathematical models that estimate reliability indexes or the power quality indexes using direct mathematical solutions. There is a widely used methodology called method of fault positions [3], which estimates the frequency of voltage sags in electrical networks. First, this approach splits the system in small parts and associates each part with a fault position. Furthermore, each one of these parts has a failure rate proportional to the length of this respective part. Second, the currents of fault for a short circuit in each fault position are evaluated using the fault analysis model. Finally, the expected frequency of voltage sags, with specified magnitude and/or duration, is evaluated. The expected values provide a valuable insight into voltage sags severity. However, there may exist a significant variability around the expected value of a probabilistic index. This variability can be assessed by generating the probability distribution functions (PDFs) of the voltage sags. The PDF

can also be used to estimate the risk of violating target values, defined by regulatory agencies, associated with voltage sags.

Usually, the PDFs associated with voltage sags are generated using the Monte Carlo simulation (MCS) [4]–[6]. The main advantage of this technique is its capability to model complex characteristics of the system that cannot be modeled by analytical approaches. The main aim of this paper is to develop a probabilistic short-circuit (PSC) approach to generate the PDF associated with the System Average RMS Variation Index (SARFI) index. The proposed methodology is based on the combination of the MCS with the admittance summation method in phase coordinates. This combination enables the following random characteristics of a fault state to be included in the generation of the SARFI PDF: fault type, phase involved in the fault, and fault position. The proposed PSC approach has been applied to generate the SARFI PDF in a distribution network of an electricity distribution company (CEMAR) in Northeast Brazil.

The rest of this paper is organized as follows. Section II establishes the methodology used to evaluate the currents of fault. Section III describes the modeling of fault random characteristics through the MCS. Section IV shows the PSC approach is used to generate the SARFI PDF. The results are depicted in Section V. General conclusions are given in Section VI.

## II. EVALUATION OF THE CURRENT OF FAULT

The evaluation of the current of fault can be carried out using both the symmetrical component method [7] and the phase coordinate method [8], [9]. In principle, the symmetrical component method cannot directly be applied to distribution systems. This constraint is due to the fact that the distribution systems are naturally unbalanced. Nevertheless, the symmetrical component method gives results with acceptable accuracy when applied to distribution networks with unbalanced load or with non-transposed lines. However, the symmetrical component method requires that the sequence circuits for each component be obtained. Moreover, it may be difficult obtain sequence circuits for some transformer connections, such as the Scott-T. On the other hand, the phase coordinate method represents explicitly all the electric network phases and their respective voltages, currents, and impedances without a need to obtain the sequence circuits. Due to this, the phase coordinate method has been used in this paper to evaluate the currents of fault.

Basically, there are four versions for the phase coordinate method used to analyze faults in distribution networks: Gauss-Zbus method [10], Kersting method [11], hybrid compensation method [12], and admittance summation method [13]. In this paper, the currents of fault were evaluated using the admittance summation method (ASM). This approach was used due to its capacity to accurately model the loads with characteristics of constant impedance and shunt susceptance. Furthermore, the ASM also has lower computational costs due to the backward-forward sweep algorithms to estimate the faults current [14].

### A. Review of the ASM

In the ASM, the nodal voltages evaluation process is done by using the backward-forward sweep, as in following [13], [14].

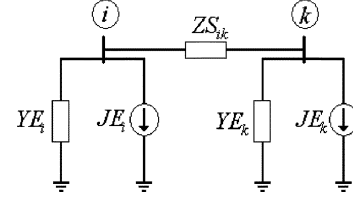


Fig. 1. Equivalent circuit used in the backward sweep.

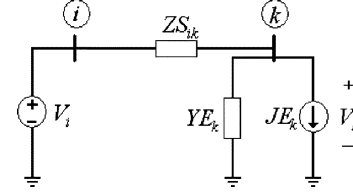


Fig. 2. Equivalent circuit for nodal voltages calculation.

#### 1) Set initial values:

$$YE_k = YL_k, \quad k = 1, \dots, n \quad (1)$$

$$JE_k = JL_k, \quad k = 1, \dots, n. \quad (2)$$

#### 2) Backward Sweep ( $k = n, \dots, 1$ ): the values $YE_k$ and $JE_k$ are evaluated using the equivalent circuit shown in Fig. 1

$$D_k = (I + ZS_{ik} \cdot YE_k)^{-1} \quad (3)$$

$$YE_i^{(new)} = YE_i^{(old)} + YE_k \cdot D_k \quad (4)$$

$$JE_i^{(new)} = JE_i^{(old)} + (I - YE_k \cdot D_k \cdot ZS_{ik}) JE_k. \quad (5)$$

In (4) and (5), the word “new” represents the equivalent after processing of the branch ( $i - k$ ), and the word “old” represents the equivalent before processing the same branch ( $i - k$ ).

#### 3) Forward Sweep ( $k = 1, \dots, n$ ): the nodal voltage values are evaluated using the equivalent circuit shown in Fig. 2

$$V_k = D_k \cdot (V_i - ZS_{ik} \cdot JE_k). \quad (6)$$

### B. Review of the Model for Primary Feeder Faults

This section presents a review of the methodology proposed in [14] to estimate the currents of faults caused by short circuit on primary feeders. When a fault occurs on the primary feeder, a new node (F) is added to the electric network after the connection of the fault admittance at a point along the primary feeder [14]. The introduction of a new node in the electric network modifies the data structures used in the backward-forward sweep algorithms. Due to this, for each simulated fault, the data structure of the backward-forward sweep algorithm must be rebuilt. Consequently, computational costs can become high in applications that require large number of faults, such as protection coordination. An alternative to overcome this problem is to eliminate the additional node introduced by the connection

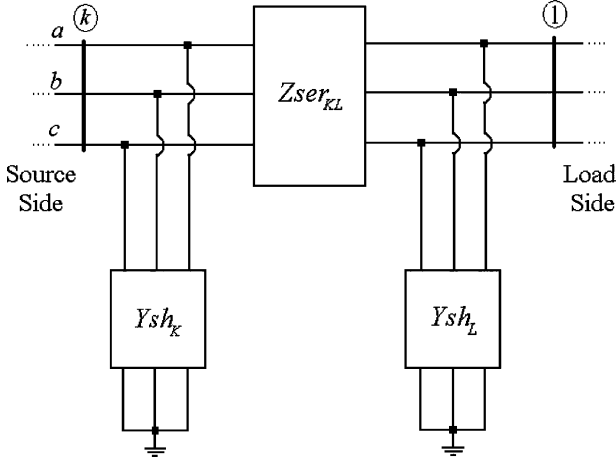


Fig. 3. Post-fault equivalent circuit.

of the fault admittance by Kron reduction method that generates a reduced equivalent circuit of the electric network. Therefore, after eliminating the fault node, the line in which a fault event has occurred can be represented by the equivalent  $\pi$  circuit shown in Fig. 3.

In this circuit,  $Z_{ser_{KL}}$  is the new primitive impedance matrix of the branch  $k-l$ . This matrix can be obtained by the following equation [12]:

$$Z_{ser_{KL}} = x(d-x) \cdot \bar{Z}_{p_{KL}} \cdot Y_{FF} \cdot \bar{Z}_{p_{KL}} \quad (7)$$

where

$$\bar{Y}_{p_{KL}} = (\bar{Z}_{p_{KL}})^{-1} \quad (8)$$

$$Y_{FF} = \frac{\bar{Y}_{p_{KL}}}{x} + \frac{\bar{Y}_{p_{KL}}}{(d-x)} + Y_f. \quad (9)$$

Due to the elimination of the fault node (F), the shunt admittances  $Y_{sh_K}$  and  $Y_{sh_L}$  are added to the nodes  $k$  and  $l$ , respectively. These shunt admittances are evaluated by the following equations:

$$Y_{sh_K} = \frac{1}{x} \cdot \bar{Y}_{p_{KL}} \cdot Y_{FF}^{-1} \cdot Y_f \quad (10)$$

$$Y_{sh_L} = \frac{1}{(d-x)} \cdot \bar{Y}_{p_{KL}} \cdot Y_{FF}^{-1} \cdot Y_f. \quad (11)$$

The entries of the fault admittance matrix  $Y_{FF}$ , in equations (7)–(11), are given in [11] for different kinds of faults.

After being evaluated in all the nodes of the post-fault network by ASM, the fault current  $I_f$  can be evaluated as follows:

$$I_f = Y_f \cdot V_F \quad (12)$$

where

$$V_F = Y_{FF}^{-1} \cdot \bar{Y}_{p_{KL}} \left[ \frac{V_K}{x} + \frac{V_L}{(d-x)} \right]. \quad (13)$$

### III. PROBABILISTIC SHORT CIRCUIT BY MONTE CARLO SIMULATION

The main advantage of the PSC analysis based on the MCS method is to provide a model that represents several aspects

TABLE I  
PROBABILITY OF SHORT-CIRCUIT OCCURRENCE

Short-Circuit	Occurrence (%)
Three-phase-to-ground	1,5
Three-phase	1,5
Line-to-line-to-ground	6,0
Line-to-line	10,0
Single-phase	81,0

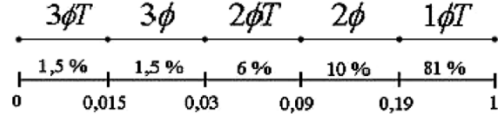


Fig. 4. Representation of the intervals that determine the type of fault.

associated with the behavior of the system, which cannot be represented through an analytical model. The MCS technique can be used in two ways: sequential or nonsequential simulation. In the sequential MCS, the system's states are sampled in chronological order. It is considered the order in which the state transitions occur. In the nonsequential MCS, the system's states are sampled randomly without considering the order in which transitions occur. In this paper, the MCS technique has been used to select the component at fault, the type and phase of fault, and fault location. As these random variables do not present chronological variations, then the nonsequential MCS has been chosen for the PSC analysis.

The PSC algorithm used in this paper to generate the SARFI PDF is summarized in the following steps.

- Step 1) Obtain the number of faults that have occurred with the selected component (feeder line section). In this paper, it has been assumed that all the components of the distribution system are operating in the useful life period. That is, the failure rate is constant. In these cases, the number of failures in a given study period follows a Poisson process [15]. In this way, the number of faults for a given component is sampled using a random number generator with Poisson distribution. The input parameters for this generator are the failure rate of the component and the study period (usually one year).
- Step 2) Select the fault location in a faulty primary feeder section using a uniform random numbers generator.
- Step 3) Select the type of fault using a uniform random numbers generator. The intervals that will determine the type of fault to be simulated are given by the probabilities of occurrence of the short-circuit types shown in Table I. Fig. 4 illustrates these intervals in detail.
- Step 4) Select the phases involved in the fault using a uniform random numbers generator. In Fig. 5 it is presented the intervals, which determine the phases involved during line-to-line faults.
- Step 5) Modify the parameters of the faulted primary feeder section using the methodologies proposed in Section II of this paper.

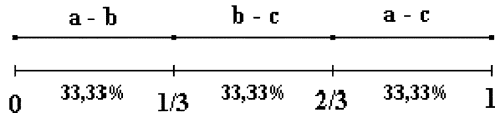
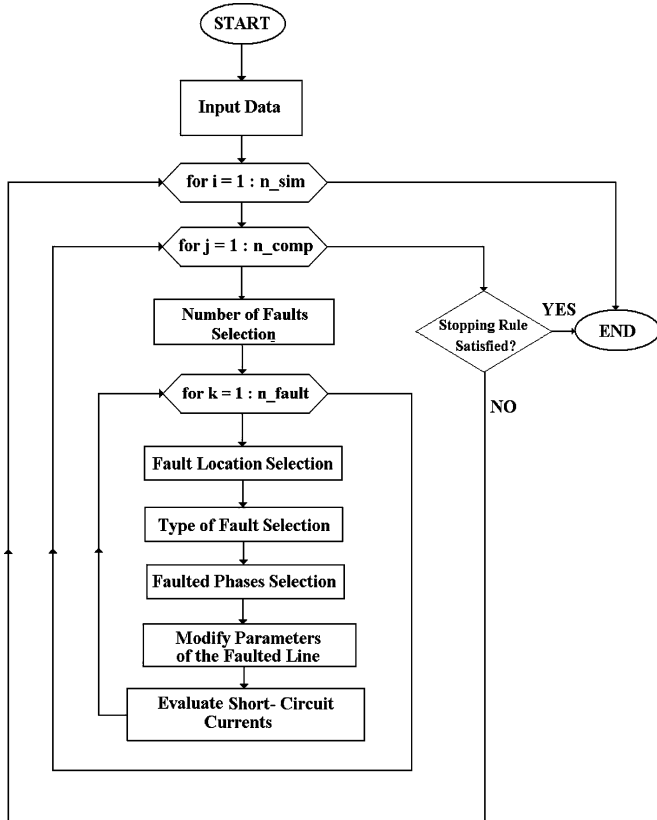


Fig. 5. Intervals for phases involved in line-to-line faults.

Fig. 6. Flow diagram of the proposed algorithm for PSC analysis, where  $n_{sim}$  is the number of simulations,  $n_{comp}$  is the number of components, and  $n_{fault}$  is the number of faults.

- Step 6) Evaluate the short-circuit current via the ASM method.
- Step 7) Repeat steps 3)–7) for the number of faults obtained in step 2.
- Step 8) Repeat steps 2)–7) for each component (primary feeder section) of the system.
- Step 9) Verify the stopping rule.
- Step 10) Repeat steps 2)–10) for the maximum number of simulations set.

Fig. 6 shows the flow diagram of the proposed algorithm to estimate the currents of fault in a radial distribution networks using the MCS method.

#### IV. APPLICATION OF THE PROPOSED METHODOLOGY IN POWER QUALITY STUDIES

The  $SARFI_x$  index represents the average number of specified RMS variation measurement events that occurred over the assessment period per customer served. The specified disturbances are those with a magnitude less than  $x$  for sags or a magnitude greater than  $x$  for swells [1].

In this new methodology for PSC evaluation, the  $SARFI_x$  for the simulation “ $i$ ” has been calculated as follows:

$$SARFI_x(S_i) = \frac{\sum_{j=1}^{NLP} f_j^x(S_i) \times N_j}{N_T} \quad (14)$$

where

- $x$  RMS voltage threshold in percentage regarding to the pre-fault scenario;
- $S_i$  simulation “ $i$ ”;
- $NLP$  number of load points;
- $f_j^x(S_i)$  number of short-duration voltage deviations with magnitudes above  $x$  percent (swells) or below  $x$  percent (sags) occurred in the load point  $j$  of the simulation  $i$ ;
- $N_j$  number of customers of the load point  $j$ ;
- $N_T$  total number of customer served from the section of the system to be assessed.

The stopping rule that has been used in the MCS method to estimate voltage deviations is the pre-specified precision. This stopping rule consists of generating fault scenarios until the relative uncertainty ( $\beta$ ) of the estimated parameter ( $SARFI_x$ ) be lower than the specified tolerance. The relative uncertainty is evaluated as follows:

$$\beta(SARFI_x) = \frac{\sigma(SARFI_x)}{E(SARFI_x) \times \sqrt{n}} \quad (15)$$

where

$$E(SARFI_x) = \frac{1}{n} \sum_{i=1}^n SARFI_x(S_i) \quad (16)$$

$$\sigma(SARFI_x) = \sqrt{\frac{1}{n-1} \sum_{i=1}^n [SARFI_x(S_i) - E(SARFI_x)]^2} \quad (17)$$

where

- $E(SARFI_x)$  mean value of the  $SARFI_x$  index;
- $\sigma(SARFI_x)$  standard deviation of the  $SARFI_x$  index;
- $n$  number of simulations set.

#### V. RESULTS

The proposed algorithm for PSC evaluation with applications in power quality analysis ( $SARFI$  evaluation) has been tested in a distribution system of Maranhão Electricity Utility (CEMAR), Brazil, shown in Fig. 7. The main characteristics of this system are presented in Table II.

In this paper, the proposed methodology has been applied to estimate the  $SARFI$  index with  $x$  equal to 10% ( $SARFI_{10}$ ), 50% ( $SARFI_{50}$ ), 70% ( $SARFI_{70}$ ) and 90% ( $SARFI_{90}$ ). These  $SARFI$  limits were obtained from [1]. The  $SARFI$  index for the test system has evaluated simulation under the following considerations:

- The fault impedance is  $1.0 + j0.0 \Omega$ .
- All the loads have been modeled with constant impedance.
- The total failure rate of the overhead primary feeders is 1.5 (failures/year/mile).

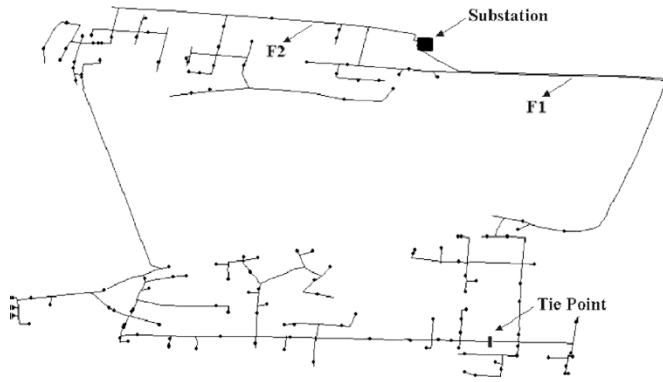
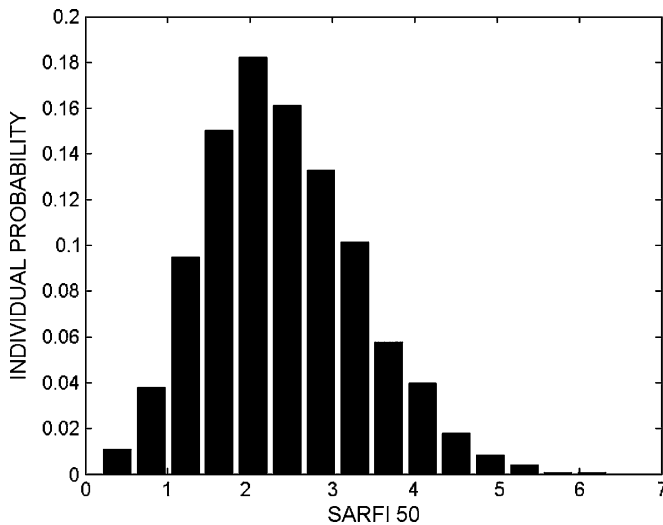


Fig. 7. Simplified online diagram of the test system.

TABLE II  
CHARACTERISTICS OF THE SYSTEM TEST

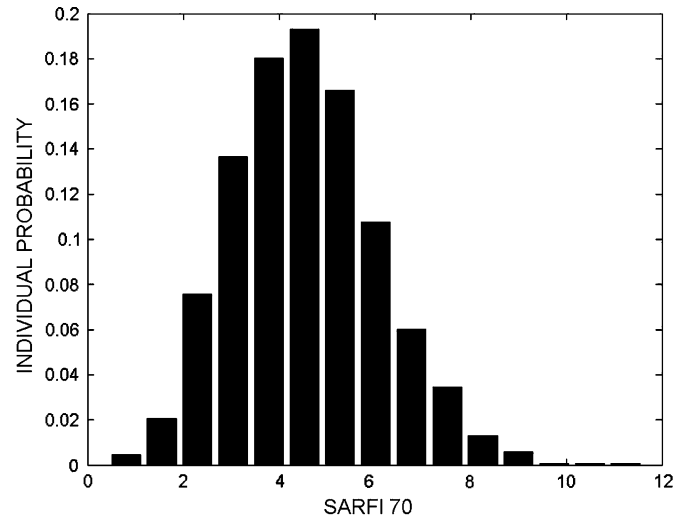
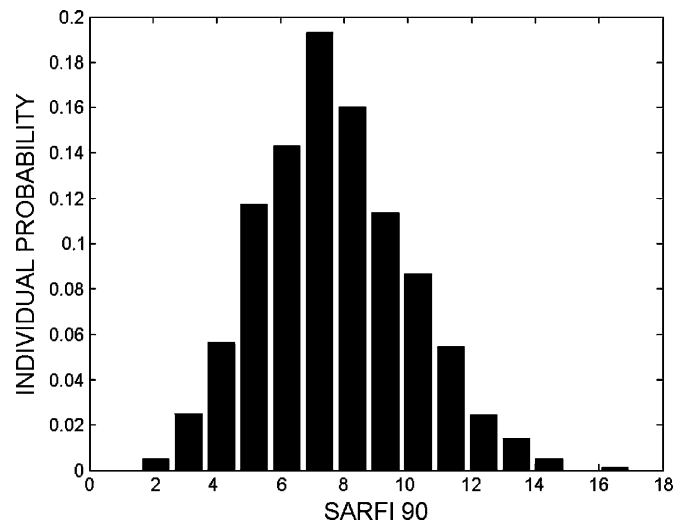
Number of Customers	4295
Peak Load [Mw]	8.545
Number of Feeders	2
Number of Load Points	138
Total Length [km]	15.683
Number of Line Sections	412
Number of Protection Devices	47
Number of Switches	19

Fig. 8. Individual probability of the SARFI<sub>50</sub>.

- The pre-specified precision used in the stopping rule is 1.0% for each estimated SARFI<sub>x</sub>.
- The study period was one year.

The total number of simulations carried out on the test system to achieve the specified precision were 1559. The execution time for these simulations is about 545.5 s (9.09 min); that means that the execution time associated with each simulation is about 349.903 ms. This execution time has been measured on a 1.6-GHz Pentium-based PC with 512 MB of RAM. These results indicate that the proposed method is computationally very efficient for applications in large distribution networks.

Figs. 8–10 show the histograms associated with SARFI<sub>50</sub>, SARFI<sub>70</sub>, and SARFI<sub>90</sub>, respectively. During the simulation with the test system, there were not any occurrences of sags of

Fig. 9. Individual probability of the SARFI<sub>70</sub>.Fig. 10. Individual probability of the SARFI<sub>90</sub>.TABLE III  
STATISTICS OF THE SARFI INDEXES

SARFI	Mean Value	Percentiles for SARFI distributions		
		25 <sup>th</sup>	50 <sup>th</sup>	75 <sup>th</sup>
50	2.3916	1.7145	2.3027	2.9996
70	4.5507	3.4814	4.4854	5.5323
90	7.6978	6.0387	7.5714	9.1374

less than 10% of the pre-fault voltage. Therefore, the SARFI<sub>10</sub> for this system is equal to zero. Table III presents some statistics associated with these indexes. From this table, it can be concluded that each customer of the test system experienced, during a yearly period, about 2.4, 4.5 and 7.7 sags smaller than 50%, 70%, and 90% of the pre-fault voltages, respectively. Table III also presents some percentiles (25th, 50th, 75th) for the SARFI distributions. The 25th percentile (first quartile) is the value that exceeds, in magnitude, one quarter (25%) of the values of the sample and is exceeded through three quarters (75%) of these values. The 50th and 75th percentile are denominated of median and third quartile, respectively. Therefore, from Table III, it can be concluded that 25% of the SARFI<sub>50</sub> values are less

than 1.7145, 50% of the SARFI<sub>50</sub> values are below 2.3027, and 75% of the SARFI<sub>50</sub> values are below 2.9996. These results demonstrated that there was a considerable variation in the SARFI expected values. Due to this, the SARFI assessments must be based on probability distributions.

## VI. CONCLUSION

This paper has described a new probabilistic short-circuit approach to generate the PDFs of the SARFI index. This technique is based on the combination of the admittance summation in phase coordinates with the nonsequential MCS method. The results have demonstrated that the proposed method is a powerful tool for power quality studies of large distribution networks.

This new methodology permits the generation of information associated with SARFI, such as mean, PDFs, and percentiles associated with these distributions. These results have demonstrated that there is a considerable variation on the SARFI's indexes around their mean values. Therefore, the SARFI index evaluation cannot be based just on considering expected values.

These results motivate the extension of the methodology to include system protection response and distributed generation resources in the power quality studies.

## REFERENCES

- [1] R. C. Dugan, M. McGranaghan, S. Santoso, and H. W. Beaty, *Electrical Power System Quality*, 2nd ed. New York: McGraw-Hill, 2002.
- [2] M. H. J. Bollen, *Understanding Power Quality Problems: Voltage Sags and Interruptions*. New York: Wiley, 2000.
- [3] *IEEE Recommended Practice for Design of Reliable Industrial and Commercial Power System*, IEEE Std. 493-1997, 1998.
- [4] S. O. Faried and A. Aboreshaid, "Stochastic evaluation of voltage sags experienced by a large industrial customer," *IEEE Trans. Power Del.*, vol. 18, no. 3, pp. 744–750, Jul. 2003.
- [5] S. Yun and J. Kim, "An evaluation method of voltage sag using a risk assessment model in power distribution systems," *Int. J. Elect. Power Energy Syst.*, vol. 25, pp. 829–839, Dec. 2003.
- [6] J. A. Martinez and J. M. Arnedo, "Voltage sags stochastic prediction using an electromagnetic transients program," *IEEE Trans. Power Del.*, vol. 19, no. 4, pp. 1975–1982, Oct. 2004.
- [7] P. M. Anderson, *Analysis of Faulted Power Systems*. New York: Wiley, 1995.
- [8] M. A. Laughton, "Analysis of unbalanced polyphase networks by the method of phase coordinates. Part I: system representation in phase frame of reference," *Proc. IEEE*, vol. 115, no. 8, pp. 1163–1172, Aug. 1968.
- [9] ———, "Analysis of unbalanced polyphase networks by the method of phase coordinates. Part II: fault analysis," *Proc. IEEE*, vol. 116, no. 5, pp. 857–865, May 1969.
- [10] T. H. Chen, M. Chen, W. Lee, P. Kotas, and P. V. Olinda, "Distribution system short-circuit analysis—a rigid approach," *IEEE Trans. Power Syst.*, vol. 7, no. 1, pp. 444–450, Feb. 1992.
- [11] W. H. Kersting, *Distribution System Modeling and Analysis*. Boca Raton, FL: CRC, 2001.
- [12] X. Zhang, F. Soudi, D. Shirmohammadi, and C. Cheng, "A distribution short circuit analysis approach using hybrid compensation method," *IEEE Trans. Power Syst.*, vol. 10, no. 4, pp. 2053–2059, Nov. 1995.
- [13] M. Todorovski and D. Rajcic, "Handling three-winding transformers and loads in short circuit analysis by the admittance summation method," *IEEE Trans. Power Syst.*, vol. 18, no. 3, pp. 993–1000, Aug. 2003.
- [14] U. Bordalo, A. B. Rodrigues, and M. G. Da Silva, "Modeling of faults on overhead lines in distribution systems using the admittance summation method," in *Proc. Int. Conf. Power System Technology*, 2004.
- [15] R. E. Brown, *Electric Power Distribution Reliability*. New York: Marcel Dekker, 2002.

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