(a) \( T_0 = 4, \quad w_0 = \frac{2\pi}{T_0} = \frac{\pi}{2} \).

Since the signal is even, \( b_m = 0 \).

\[
\begin{align*}
\alpha_0 &= \frac{3}{T_0} \int_{-1}^{3} x(t) dt = \frac{4}{T_0} \left( \int_{-1}^{1} dt - \int_{1}^{3} dt \right) = 2 - 2 = 0 \\
\alpha_n &= \frac{2}{T_0} \int_{-2}^{2} x(t) \cos \left( \frac{n\pi}{2} t \right) dt \\
&= \frac{4}{T_0} \int_{0}^{2} x(t) \cos \left( \frac{n\pi}{2} t \right) dt \\
&= \int_{0}^{1} \cos \left( \frac{n\pi}{2} t \right) dt - \int_{1}^{2} \cos \left( \frac{n\pi}{2} t \right) dt \\
&= \frac{2}{n\pi} \left[ \sin \left( \frac{n\pi}{2} \right) \right]_{t=0}^{t=1} - \frac{2}{n\pi} \left[ \sin \left( \frac{n\pi}{2} t \right) \right]_{t=1}^{t=2} \\
&= \frac{2}{n\pi} \left( \sin \left( \frac{n\pi}{2} \right) - 0 \right) - \frac{2}{n\pi} \left( 0 - \sin \left( \frac{n\pi}{2} \right) \right) \\
&= \frac{4}{n\pi} \sin \left( \frac{n\pi}{2} \right)
\end{align*}
\]

(b) \( T_0 = 2\pi, \quad w_0 = \frac{2\pi}{T_0} = 1 \).

\[
\begin{align*}
\alpha_0 &= \frac{1}{10} \int_{0}^{2\pi} x(t) dt = \frac{1}{2\pi} \int_{0}^{2\pi} \frac{1}{2\pi} t dt = \frac{1}{4\pi^2} \cdot \frac{1}{2} \left[ \frac{2\pi}{2} \right] \quad = \frac{1}{2} \\
\alpha_n &= \frac{2}{T_0} \int_{0}^{2\pi} x(t) \cos (nt) dt = \frac{2}{2\pi} \int_{0}^{2\pi} \frac{1}{2\pi} \cos (nt) dt \\
&= \frac{1}{2\pi} \left[ \int_{0}^{2\pi} \frac{1}{2\pi} \cos (nt) dt \right]_{t=0}^{t=2\pi} \\
&= \frac{1}{2\pi^2} \left[ \cos (2n\pi) + 2n\pi \sin (2n\pi) - \cos (0) - 0 \right] \\
&= \frac{1}{2\pi^2} \left[ 1 + 0 - 1 - 0 \right] = 0
\end{align*}
\]
6.1-1. (c) \( b_n = \frac{2}{T_0} \int_0^{2\pi} x(t) \sin(nt) \, dt = \frac{2}{2\pi} \int_0^{2\pi} \frac{1}{2\pi} \sin(nt) \, dt \)

\[ = \frac{1}{2\pi^2} \int_0^{2\pi} t \sin(nt) \, dt \]

\[ = \frac{1}{2\pi^2} \cdot \frac{1}{n^2} [\sin(nt) - nt \cos(nt)] \bigg|_0^{2\pi} \]

\[ = \frac{1}{2n^2\pi^2} \left[ \sin(2n\pi) - 2n\pi \cos(2n\pi) - \sin(0) + 0 \right] \]

\[ = \frac{1}{2n^2\pi^2} \left[ 0 - 2n\pi - 0 + 0 \right] \]

\[ = -\frac{1}{n\pi} \]

(d) \( T_0 = \pi \), \( \omega_0 = \frac{2\pi}{T_0} = 2 \).

Since the signal is odd, \( a_0 = 0 \), \( a_n = 0 \)

\( b_n = \frac{2}{T_0} \int_{-\pi/2}^{\pi/2} \frac{4}{\pi} t \sin(2nt) \, dt = \frac{4}{\pi} \int_{-\pi/2}^{\pi/2} \frac{4}{\pi} t \sin(2nt) \, dt \)

\[ = \frac{16}{\pi^2} \int_0^{\pi/2} t \sin(2nt) \, dt = \frac{16}{\pi^2} \cdot \frac{1}{4n^2} [\sin(2nt) - 2nt \cos(2nt)] \bigg|_0^{\pi/2} \]

\[ = \frac{4}{n^2\pi^2} \left[ \sin(n\pi) - \frac{n\pi}{2} \cos(n\pi) - 0\sin(0) + 0 \right] \]

\[ = \frac{4}{n^2\pi^2} \left[ \sin(n\pi) - \frac{n\pi}{2} \cos(n\pi) \right] \]
6.3-1 (a) \( T_0 = 4, \omega_n = \frac{2\pi}{T_0} = \frac{\pi}{2}, \chi(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\frac{\pi}{2} t} \)

\[
D_n = \frac{1}{4} \left( \int_{-1}^{1} e^{-jn\frac{\pi}{2} t} dt - \int_{-1}^{1} e^{-jn\frac{\pi}{2} (t+2)} dt \right)
\]

For \( n = 0, \) \( D_0 = \frac{1}{4} \left( \int_{-1}^{1} dt - \int_{-1}^{1} dt \right) = \frac{1}{4} (2 - 2) = 0. \)

For \( |n| > 1, \) \( D_n = \frac{1}{4} \left[ \frac{2}{jn\pi} e^{-jn\frac{\pi}{2} t} \right]_{t=-1}^{t=1} + \frac{2}{jn\pi} e^{-jn\frac{\pi}{2} t} \right]_{t=1}^{t=3} \)

\[
= \frac{1}{2jn\pi} \left( e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}} + e^{-jn\frac{\pi}{2}} - e^{jn\frac{\pi}{2}} \right)
\]

\[
= \frac{1}{2jn\pi} \left( e^{jn\frac{\pi}{2}} - 2e^{-jn\frac{\pi}{2}} + e^{jn\frac{\pi}{2}} \right)
\]

\[
= \frac{1}{2jn\pi} \left( e^{jn\frac{\pi}{2}} - 2e^{-jn\frac{\pi}{2}} + e^{jn\frac{\pi}{2}} \right)
\]

\[
= \frac{2}{jn\pi} \left( e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}} \right)
\]

\[
= \frac{2}{jn\pi} \sin\left(\frac{n\pi}{2}\right)
\]
(0.3-1) \quad T_0 = 2\pi , \quad \omega_0 = \frac{2\pi}{T_0} = 1

\begin{align*}
D_0 &= \frac{1}{T_0} \int_{T_0} x(t) dt = \frac{1}{2\pi} \int_0^{2\pi} \frac{t}{2\pi} dt \\
&= \frac{1}{2\pi} \left[ \frac{t^2}{4\pi} \right]_{t=0}^{t=2\pi} = \frac{4\pi^2}{8\pi^2} - 0 = \frac{1}{2}
\end{align*}

\begin{align*}
&\text{for } n \neq 1, \quad D_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-j n \omega_0 t} dt \\
&= \frac{1}{2\pi} \int_0^{2\pi} \frac{t}{2\pi} e^{-j nt} dt \\
&= \frac{1}{4\pi^2} \int_0^{2\pi} t e^{-j nt} dt
\end{align*}

Since \( \int t e^{at} dt = \frac{e^{at}}{a^2} (at - 1) \)

\begin{align*}
D_n &= \frac{1}{4\pi^2} \left( \frac{e^{-j nt}}{-j n^2} \right) (-j nt - 1) \bigg|_{t=0}^{t=2\pi} \\
&= \frac{1}{4\pi^2} \left( \frac{e^{-j nt} (1 + j nt)}{-n^2} \right) \bigg|_{t=0}^{t=2\pi} \\
&= \frac{1}{4\pi^2} \left[ \left( e^{j 2\pi t} (1 + j 2\pi t) \right) - 1 \right]
\end{align*}

And

\[ e^{j 2\pi t} = \cos(-2\pi t) + j \sin(-2\pi t) = 1 \]

Thus

\[ D_n = \frac{1}{4\pi^2} \left( 1 + j 2\pi t - 1 \right) \]

\[ = \frac{j}{2\pi} \]
\( T_0 = \pi, \quad \omega_0 = \frac{2\pi}{T_0} = 2. \)

\[
D_n = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} \ e^{-j2\pi nt} \ dt.
\]

For \( n = 0 \), \( D_0 = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} \ dt = \frac{4}{\pi^2} \ \frac{\pi^2}{2} \ \left| t = \frac{\pi}{4} \right| = 0. \)

For \( |n| > 1 \), \( D_n = \frac{4}{\pi^2} \int_{-\pi/4}^{\pi/4} t \ (-\frac{1}{j2n}) e^{-j2\pi nt} \ dt \)

\[
= -\frac{2}{jn\pi^2} \left[ \left. te^{-j2\pi nt} \right|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - \left( \frac{\pi}{4} e^{-j2\pi nt} \right) \right] - \frac{2}{jn\pi} \left[ \pi e^{-j\pi(\frac{n}{2})} + \pi e^{j\pi(\frac{n}{2})} + j\pi e^{-j2\pi nt} \right] \]

\[
= -\frac{2}{jn\pi^2} \left[ \left. \frac{n}{2} \cos\left(\frac{n\pi}{2}\right) + \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) \right] \]

\[
= \frac{2}{n\pi^2} \left[ \frac{n}{2} \cos\left(\frac{n\pi}{2}\right) - \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) \right] \]