Unit Five Quiz Solutions and Unit Six Goals

Mechanical Engineering 370
Thermodynamics

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Outline

- Quiz Five Solutions
  - Use open system equations
  - Use appropriate property relations
- Unit six – first law for complex systems
  - Consider unsteady processes with mass flow across boundary
  - Start with first law for open systems
  - Use average properties across boundaries and integrated heat and work effects

Quiz Five Solution

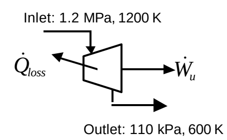
- Given: Ideal gas, R = 0.3 kJ/kg·K, $c_p = a + bT$
  - T_{in} = 1200 K, P_{in} 1.2 MPa, T_{out} = 600 K, P_{out} = 110 kPa, Heat Loss = 1.5 kW, Power out = 500 kW
- Find the mass flow rate
- Start with the general first law and mass-balance equations

$$\frac{dm_{system}}{dt} = \sum_{inlet}\dot{m}_i - \sum_{outlet}\dot{m}_i$$

$$\frac{dE_{system}}{dt} = \dot{Q} - \dot{W}_u - \sum_{outlet}\dot{m}_i\left(h_i + \frac{\vec{V}_i^2}{2} + gz_i\right) + \sum_{inlet}\dot{m}_i\left(h_i + \frac{\vec{V}_i^2}{2} + gz_i\right)$$

Quiz Five Solution Continued

- One inlet and one outlet
- Assume steady-state
- Neglect changes in kinetic and potential energies



Finding the Answer

$$\dot{Q} = \dot{W}_u + \dot{m}(h_{out} - h_{in}) \quad \Rightarrow \quad \dot{m} = \frac{\dot{Q} - \dot{W}_u}{h_{out} - h_{in}}$$

$$h_{out} - h_{in} = \int_{T_{out}}^{T_{in}} c_p\,dT = \int_{T_{out}}^{T_{in}} (a + bT)\,dT = a(T_{out} - T_{in}) + \frac{b\left(T_{out}^2 - T_{in}^2\right)}{2} =$$

$$0.9\frac{kJ}{kg \cdot K}(600K - 1200K) + \frac{0.0003}{2}\frac{kJ}{kg \cdot K^2}\left[(600K)^2 - (1200K)^2\right] = -702\frac{kJ}{kg}$$

$$\dot{m} = \frac{\dot{Q} - \dot{W}_u}{h_{out} - h_{in}} = \frac{-1.5\,kW - 500kW}{-702\,kJ/kg}\frac{kJ \cdot s}{1\,kW} = 0.714\frac{kg}{s}$$

Unit Six Goals

- As a result of studying this unit you should be able to
  - understand the meaning of individual terms in the first law and mass balance for unsteady flows
  - apply the first law and mass balance equations to solve problems in unsteady flows

Unsteady Flow Terms

$m_i = \int \dot{m}_i dt$ Integrated mass flow (kg)

$Q = \int \dot{Q} dt$ Integrated heat transfer (kJ)

$W_u = \int \dot{W}_u dt$ Integrated useful work (kJ)

$\langle h_i \rangle = \dfrac{\int \dot{m}_i h_i dt}{\int \dot{m}_i dt}$ Average stream property

More Unsteady Flow Terms

Change in system energy during process (kJ)

$$\Delta E_{system} = E_2 - E_1 = \int \frac{dE_{system}}{dt} dt$$

Change in system mass during process (kg)

$$\Delta m_{system} = m_2 - m_1 = \int \frac{dm_{system}}{dt} dt$$

Unsteady Flow Equations

$$\left[m_2 \left(u + \frac{\vec{V}^2}{2} + gz \right)_2 - m_1 \left(u + \frac{\vec{V}^2}{2} + gz \right)_1 \right]_{system} = Q - W_u$$

$$- \sum_{outlet} m_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + \sum_{inlet} m_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

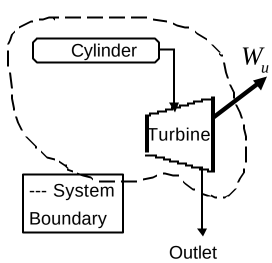
$$[m_2 - m_1]_{system} = \sum_{inlet} m_i - \sum_{outlet} m_i$$

Example Calculation

- Given: A compressed air cylinder with a volume of 1.5 ft³ is used to power a turbine.
- State and flow data
  - T_1 = 520 R, P_1 180 psia
  - T_2 = 460 R, P_2 = 120 psia
  - Turbine outlet P_{out} = 110 kPa, T_{out} = 360 R
- Negligible heat transfer
- Find turbine work

Diagram and Assumptions

- Define system as cylinder and turbine
- Only one outlet
- Negligible kinetic and potential energy changes
- Q = 0 (given)



Get the First Law for Problem

$$\left[m_2 \left(u + \frac{\vec{V}^2}{2} + gz \right)_2 - m_1 \left(u + \frac{\vec{V}^2}{2} + gz \right)_1 \right]_{system} = Q - W_u$$

$$- \sum_{outlet} m_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + \sum_{inlet} m_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

$$[m_2 u_2 - m_1 u_1]_{system} = -W_u - m_{out} h_{out}$$

Mass Balance plus First Law

$$[m_2 - m_1]_{system} = \sum_{inlet} m_i - \sum_{outlet} m_i = -m_{out}$$

$$[m_2 u_2 - m_1 u_1]_{system} = -W_u - m_{out} h_{out}$$

$$[m_2 u_2 - m_1 u_1]_{system} = -W_u + [m_2 - m_1]_{system} h_{out}$$

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Use Ideal Gas with Air Tables

- R = 0.06855 Btu/lb$_m$·R from Table A-1E, page 874
- 1 Btu = 5.40395 psia·ft^3
- From Table A-17E, page 896
  - u_1 = u(520 R) = 88.62 Btu/lb$_m$
  - u_2 = u(460 R) = 78.36 Btu/lb$_m$
  - h_{out} = h(360 R) = 85.97 Btu/lb$_m$

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Use PV = mRT for m_1 and m_2

$$m_1 = \frac{P_1 V_{cyl}}{R T_1} = \frac{(180\ psia)(1.5\ ft^3)\dfrac{1\ Btu}{5.40395\ psia\cdot ft^3}}{(500\ R)\dfrac{0.06855\ Btu}{lb_m \cdot R}} = 1.458\ lb_m$$

$$m_2 = \frac{P_2 V_{cyl}}{R T_2} = \frac{(120\ psia)(1.5\ ft^3)\dfrac{1\ Btu}{5.40395\ psia\cdot ft^3}}{(460\ R)\dfrac{0.06855\ Btu}{lb_m \cdot R}} = 1.056\ lb_m$$

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Get the Answer

$$W_u = [m_1 u_1 - m_2 u_2]_{system} + [m_2 - m_1]_{system} h_{out} =$$

$$(1.458\ lb_m)\left(88.62\ {}^{Btu}\!/_{lb_m}\right) - (1.056\ lb_m)\left(78.97\ {}^{Btu}\!/_{lb_m}\right)$$

$$+ (1.056\ lb_m - 1.458\ lb_m)\left(85.97\ {}^{Btu}\!/_{lb_m}\right) = 11.26\ Btu$$

- Positive work is a net work output. Work is ~0.0044 hp-hr or ~0.0033 kWh

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