## Lecture <br> Chapter 10

## Algorithm Efficiency \& Sorting

## Measuring Algorithm Efficiency

- not coding, i.e., implementation
- not platforms, i.e., computer systems
- not the data sets
- but the execution time of the algorithm as measured by the number of operations required by the algorithm

Algorithm $A$ is of the order $f(n)$, i.e., $O(f(n))$
If there exists constants $k$ \& $n_{0}$ such that
A produces a solution to a problem of size $n>=n_{0}$ within $k$ * $f(n)$ time units
e.g., $f(n)=n^{2}-3^{\star} n-10$
if $k=3 \& \mathbf{n}_{\mathbf{0}}=2$ then
for all $\mathrm{n}>=2$

$$
\underline{3}^{*} n^{2}>n^{2}-3^{*} n-10
$$

hence $f(n)$ is of order $O\left(n^{2}\right)$
e.g., Linked List
displaying/searching the first $\mathbf{n}$ items requires
$(n+1)^{\star}(a+c)+n^{\star} w$ time units
for $n>=1$

$$
\underline{\left(2^{*} a+2^{*} c+w\right)^{*} n}>=(n+1)^{*}(a+c)+n^{*} w
$$

hence the task is of order $O(n)$
e.g., Towers of Hanoi
solution requires $\left(2^{n}+1\right)$ * $m$ time units
for $n>=1$
$\underline{m}^{*} 2^{n}>\left(2^{n}+1\right) * m$
hence the solution is $O\left(2^{n}\right)$
Order of Growth Rates

$$
O(1)<O(\log n)<O(n)<O\left(n^{*} \log n\right)<O\left(n^{2}\right)<O\left(n^{3}\right)<O\left(2^{n}\right)
$$

- largest order term absorbs smaller order terms
- multiplicative and additive constants are absorbed by higher order terms
- $O(f(n))+O(g(n))=O(f(n)+g(n))$


## Worst Case Analysis

## Average Case Analysis

An application's

- structure
- size
- execution time requirements
- memory size requirements
will often dictate the appropriate solution
algorithms used to solve large problems $\rightarrow$ order of magnitude analysis

